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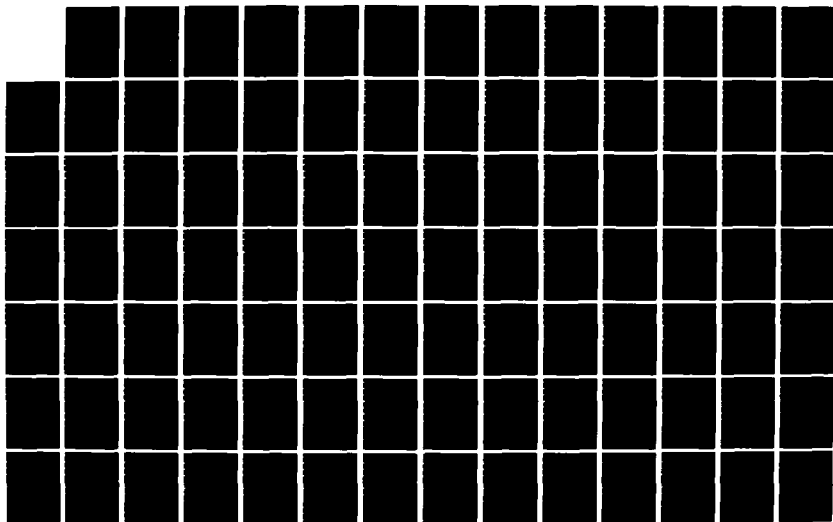
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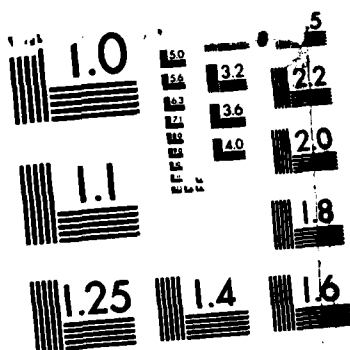
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For the degree of Master of Science

Signed by the final examining committee:

Jane M. Fraser, chair
Donald T. Frazier
Dan Koenig

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Jane M. Fraser
Major professor

**TOWARD AN EXTENSION OF DECISION ANALYSIS TO
COMPETITIVE SITUATIONS**

A Thesis

Submitted to the Faculty

of

Purdue University

by

Steven Dean Knott

**In Partial Fulfillment of the
Requirements for the Degree**

of

Master of Science

December 1985

To Mom, Dad, and Julie

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ABSTRACT

Knott, Steven Dean, Capt. USAF. M.S., Purdue University, December 1985.
Toward an Extension of Decision Analysis to Competitive Situations. Major
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→ The purpose of this ^{thesis} ~~research~~ is to lay the foundation for the extension of Decision Analysis to enable it (1) to advise a decision maker in a competitive situation and (2) to model the preferences of decision makers who do not accept the axioms of Decision Analysis. The intention is to combine Game Theory with Decision Analysis to accomplish the first goal. The second goal will be achieved through the use of utility models currently under development by several investigators.

A unified methodology is presented. It contains three major parts: the model of the game, a typology of possible players, and a solution method. The model is the Decision Analysis decision tree, modified to include competitors. The typology of players is a modification of a proposal by Harsanyi, [1957-8], and contains the decision maker's information on his opponents. The solution method itself has three major parts. The first is the use of a modified form of rationalizable solutions (~~[Bernheim, 1984]~~ and ^{or} [Pearce, 1984]) as the

Cont → basic decision method. The second is the use of Bayesian updating to gain information on the type of an opponent based on his previous moves. The third is the use of Hypergames ~~(Bennett, 1977)~~ as an additional method of sensitivity analysis to aid in correctly analyzing the infinite regress of expectation.

The proposed methodology is mathematically equivalent to treating the opponents' moves as random events. However, the methods by which we assess these probabilities and conduct sensitivity analysis should make this model much more accurate than direct assessment.

↑

CHAPTER I

INTRODUCTION

A. INTRODUCTION

The primary goal of this thesis is to develop a methodology to extend Decision Analysis to deal with decisions where the actions of competitors have a direct impact on the payoff received by the decision maker. This requires that we model and analyze the actions of persons other than the decision maker we are advising. A second goal is to model the preferences of decision makers which cannot be modeled by expected utility. This thesis does not accomplish these goals, but it does outline a structure for their accomplishment. It is a report of research in progress and an outline for further research. We assume knowledge of the basics of Decision Analysis, Utility Theory, and Game Theory.

Others are working on methods to analyze real world competitive situations, Fraser and Hipel [1984], Bennett and Huxham [1982], but they base their work on Game Theory, rather than Decision Analysis as we have. Others have also been working on the issue of expected utility, Allais [1979], Bell [1982 and 1985], and Machina [1983] to name a very few. We intend to make extensive use of the ideas of these people in order to move toward our common objectives.

The remainder of this chapter gives our philosophy, describes and justifies our specific goals, and introduces the general form of the proposed methodology. Chapter II explains and justifies modifications to the usual decision tree. Chapter III covers a proposed typology of players to be used with the model. Chapter IV explores solution methods and sensitivity analysis. Chapter V concludes and summarizes the thesis and some specific applications issues.

B. GOALS AND PHILOSOPHY

Our primary objective is to develop a methodology to advise people faced with difficult decisions in competitive situations. We call this proposed methodology Extended Decision Analysis (XDA). This methodology could be viewed either as adding competition to Decision Theory or as making Game Theory prescriptive, since it is obvious that elements of each will be necessary. To accomplish this we combine Decision Analysis and Game Theory to

form a prescriptive theory for advising a specific decision maker facing active competition. We choose to base our methodology on Decision Analysis rather than Game Theory because it is already equipped to analyze situations which, by their very nature, have incomplete information and Decision Analysis is focused on the objective of advising a single decision maker.

Some of the changes we make in Decision Analysis in order to deal with competition may ease the use of non-von Neumann-Morgenstern utility. This leads to our secondary goal of questioning expected utility as the sole prescriptive basis for rational decision making. We intend to adapt the work of others and to develop our own methods of dealing with departures from the standard axioms of Decision Analysis.

The final goal is to combine all of these factors into a coherent, well reasoned methodology that can be easily understood by the decision maker and easily implemented by the analyst. This specifically includes the design of tree notation and the presentation of the data derived from analysis of the model. This is the engineering goal as opposed to the two previous theoretical goals.

Two general philosophies underlie our efforts, one dealing with the relationship between XDA and the decision maker and the second dealing with the structure of the model itself. The primary goal of XDA is to aid the decision maker in making consistent and logical decisions based on his knowledge and the assumptions which he chooses to make. We make no judgments on

the normative value of the various possible assumptions. Although we realize that an analyst has a certain responsibility to state his considered opinions as to what has normative value and what does not, we are not in that position. We are in the position of designing and building an analytical tool of sufficient capability to allow the analyst to assist the decision maker. This leads us directly to the philosophy of the methodology itself. This is a philosophy of flexibility. The goal is that this methodology will provide techniques for carrying out a logical analysis of the game using all available data and any reasonable set of assumptions which the decision maker and his analyst care to make.

C. JUSTIFICATION OF THE COMPETITION GOAL

Why is it necessary to add the ability to deal with competition to Decision Analysis? Decision makers are faced constantly with difficult decisions in which the actions of competitors must be taken into account. There is at present no suitable methodology for advising decision makers faced with such problems.

There have been attempts to apply Decision Analysis to situations in which competition plays a large part but they have not been very successful. In Decision Analysis the only method for accounting for actions other than the decision maker's is through a chance node. Bell [1984] attempted such an analysis with disappointing results. While von Winterfeldt [1980] attempted a

more detailed analysis using three separate decision trees, one for each side in the dispute, he felt that this still failed to capture the interactive nature of decision making with competition.

Game theory poses a different problem. It can analyze games with both competitor and chance moves, but it does not aim to aid one particular player. Its emphasis is on finding a solution that is mutually prescriptive for all of the players. In addition, only recently has there been much work addressing the situation where the players are not informed of the complete structure of the game, that is, games of incomplete information [Harsanyi, 1967-8]. These criticisms apply with nearly equal force to practical analysis techniques based on Game Theory. None of them provides a practical framework to aid one decision maker in making one decision.

For a more complete analysis of this issue see Rees [1984] and Fraser and Rees [1985].

D. JUSTIFICATION OF THE UTILITY GOAL

Many researchers have become convinced that the deviations of actual individuals from the dictates of Expected Utility Theory are significant and require some modification of that theory. We agree that there are considerations of normative value which are not adequately addressed by Expected Utility.

There is a great deal of evidence that people do not make their decisions in accordance with the tenets of Expected Utility [Allais, 1979]. There is indeed further evidence that these deviations from Expected Utility are systematic and predictable [Machina, 1983]. Indeed Blatt [1983], though somewhat strident in his tone, has demonstrated a valid class of rational persons whose actions are both rational and inconsistent with Expected Utility Theory.

The substitutability or independence axiom is the axiom most often attacked [Machina, 1983] [Munera and de Neufville, 1983] as the cause of these inconsistencies. The hidden implication of this axiom is that preferences vary linearly with probabilities. Psychologists have found that this is not the case [Tversky and Kahneman, 1981]. Actual preferences may vary with changes in the shape of the distribution given a constant expected value [Lopes, 1984]. For over 200 years it has been more or less accepted that preferences on value are not linear, and that these preferences are both valid and rational; why should it be difficult to accept that the same is true for probabilities?

Loomes and Sugden [1984] have attacked the validity of both the sure thing principle and the transitivity axiom. They argue that lotteries should be viewed as "bundles of goods" and thus the quantity of possible outcomes, even if unrealized (as we are speaking of a probabilistic event), has an effect on the valuation of other outcomes, specifically the one which actually occurs.

Similar arguments can be made to support normative value of the concepts of regret and disappointment [Bell, 1982, 1985] [Hagen, 1984].

On another line, Pope [1984] has made an effort to resolve the problem of the "Utility of Gambling." This concept of process oriented utility is both illuminating and useful, if for no other reason than to aid persons who, when faced with an important decision, do not want to be influenced by it. As a practical matter of application to Decision Analysis however it seems to us that the same information can be captured by a utility function modeled after Ralffa's π -BRLT system [1968], where the utility at the tip of the tree can be expressly defined as containing all feelings about the path used to reach that tip. Such a utility function would have, however, no meaning when separated from the tree and would make it very difficult to break out and eliminate unwanted influences on the utility function.

There thus exists sufficient doubt as to the normative value of the Expected Utility model that any methodology which purports to advise decision makers must be able to accommodate those persons who find some or all of the axioms of Expected Utility to be unacceptable. We see our function as advising the decision maker so that he can make reasoned, logical, and consistent decisions based on his beliefs, not as prescribing his beliefs for him.

E. PROPOSED METHODOLOGY

The basis of our proposed methodology is a modification of Harsanyi's [1967-68] method of dealing with games of incomplete information. Harsanyi proposed modeling a game of incomplete information as one in which players, instead of being unaware of certain information about their opponents, are unaware of the specific "type" of those opponents. The players do, however, have a subjective probability distribution over the types for each player. The major assumption of the theory is that this distribution is commonly held by all of the players involved in the game. Thus the game can be modeled as beginning with a chance move which resolves the types of the players in such a way that each player knows only his own type. In this form the game can be reduced to normal form by taking expectation over this node and solved using appropriate procedures.

The modifications we apply to this model are very extensive as we are imposing this Game Theory concept on what is basically a Decision Theoretic framework. Since we have only the information available to one player we do not need to assume that all players hold the same distribution. For reasons discussed in Chapter II we conduct most of our calculations in extensive form. Thus we do not model the subjective distribution over opponent types as a chance node, but as a repository of the information which the decision maker has on that opponent. The typology presented in Chapter III aids the analyst in eliciting that information from the decision maker. Our typology is loosely

organized and flexible emphasizing word pictures easily understood by the decision maker. This typology also includes definitions of the several possible forms of the utility function. To further increase the flexibility of this concept we use a modification of the concept of rationalizable strategies (Bernheim [1984] and Pearce [1984]) as the major solution method for opponent decision nodes.

Rationalizable strategies are a mathematically well defined concept closely related to the iterative deletion of strongly dominated strategies. Bernheim and Pearce rejected the usual notion of equilibrium as rationality and returned to the basis of "rational" thought as defined in economics, Savage's axioms. We modify this concept by arguing first, with March [1978], that there are valid forms of rationality which are not related to calculated rationality let alone the specific form of calculated rationality implied by Savage's axioms. Secondly, even if this is not so, in order to effectively aid our decision maker we must be able to analyze all forms of rationality and even systematic irrationality since these could be used by our opponents. The immediate corollary is that what is a rationalizable strategy depends on one's type of rationality. Thus our form of rationalizable strategies includes the original concept and indeed every Game Theoretic solution concept as a subset.

The last major new concept in our methodology is the least developed at this point in time, and has the most promise for increasing the effectiveness of

Decision Analysis. We make two attempts to improve the analysis of the infinite regress of expectation, problems of the form "I think that he thinks that I think...."

In a sequential game a player's moves reveal information on his actual type. This information should modify the decision maker's subjective distribution of that player's type according to Bayes' Rule, but this calculation leads to a direct confrontation with the infinite regress of perception. We have identified certain structures for which these calculations are to be made.

The concept of hypergames proposed by Bennett [1977] and others is a method of analyzing in a systematic manner the effects of misperceptions by players of each other's utility functions. In our framework we are, of course, limited to the information available to a single player; thus a formulation of the hypergame is a statement of the infinite regress. The procedure should be easily modified to examine misperceptions other than those about utility, and could yield excellent results.

CHAPTER II

THE TREE

A. FORMAT AND THE MODEL

1. General Model

A real world competitive situation involves incomplete information since the only information which we will have available is that which the decision maker can provide us. In order to handle this we propose to use a modification of the model proposed by J. C. Harsanyi [1967-8], to deal with Bayesian games of incomplete information.

In Harsanyi's model the uncertainty of one player about another is modeled as uncertainty about that player's type. A chance node at the beginning of the game determines the players' types in such a way that although each player is aware of his own type he is unaware of his opponents'. Each

player is considered to have a subjective distribution on the types of his opponents; in Harsanyi's model this distribution is commonly held by all of the players. Thus the game trees as described below can be visualized as following this chance node which is not explicitly shown in our model.

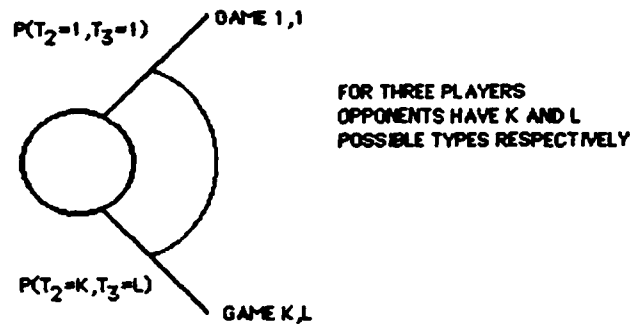


Figure 1
Harsanyi Model

We avoid the objections raised by Aumann and Maschler to games with an initial chance move as we intend to conduct the majority of the analysis in extensive form. In fact, as the original purpose of this model was to allow a game of incomplete information to be reduced to strategic form, we can do away with this initial chance node altogether. Thus we can view the single subjective distribution we have access to, the decision maker's, as a belief structure imposed on the game tree rather than a part of the tree itself. This allows us to avoid objections to the use of tree dependent structural utilities,

derived on the game tree, in what would be a different and much larger tree. In addition it allows us to treat this distribution as a prior and apply Bayesian updating, based on the information gained from the moves made by other players, as part of our solution concept.

This model should allow us to deal with our decision maker's information and judgments about his opponents in a well defined manner, provided that we have a practical typology of players as a framework to base this information on, and a solution method which takes this information into account.

2. The Game Tree

Our game tree will be a combination of the forms used in Game Theory and Decision Analysis, with Decision Analysis predominating. The modifications which we will employ are those which are necessary in order to effectively model a competitive situation. We will use three types of nodes to construct our extensive trees. As an example consider figure 2 which is a representation of a decision on whether or not to take your umbrella when going on a walk with a friend.

There will be two types of decision nodes. Those representing decisions made by our decision maker are shown as squares, labeled with the number one and subscripted to indicate information set. In our example this is the first node, whether or not to take the umbrella. Those representing decisions made by other players in the game are shown as diamonds, labeled by

numbers 2 through n to designate which player, the labels are subscripted to indicate information sets. In the example this is our friend's decision on whether or not to take his umbrella; note that he makes his decision without knowing the result of our decision. Each branch from a decision node has a description of the decision associated with it.

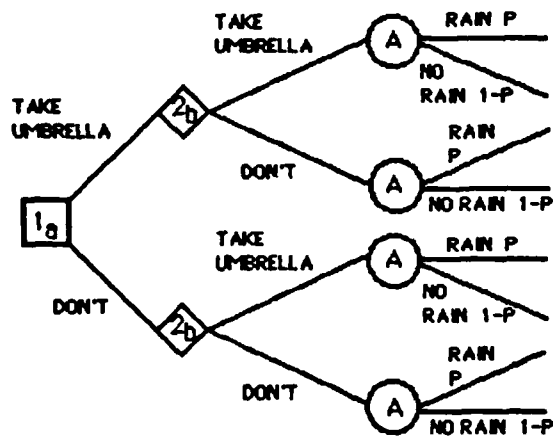


Figure 2
Umbrella Game

The notation we use on the two types of decision nodes is different for two reasons. The first reason is that in our model the two types of nodes are handled in inherently different ways and mean very different things. The second reason is that we want to emphasize this difference in an easily understood and perceived manner, i.e., to facilitate visual presentation of the tree. We are not changing notation with regard to information sets just to be

different; information acquisition under this structure is not as well defined as in standard Game Theory as the game itself is not specifically defined. In addition we are attempting to take into account at a specific decision node the effect of knowledge of events which are not predecessors of that node. Thus we require a more general notation for the information state of a player. The concept of information horizon, which is under development and should provide a useful tool in describing this situation, will probably use notation similar to the standard game theory information set. The information horizon may be a concept similar to a shared information set for all players, a time horizon of common information. Alternately, it may be necessary to have a separate information horizon structure for groups of players or even individual players. This may be a substitute for or in conjunction with a common information structure.

Chance nodes will be designated by a circle which is labeled with a letter, to show a specific stochastic process, which may be subscripted with a letter if the node is not independent of other chance nodes, and/or with a number if it is conditional on a preceding decision node. For example in figure 2 the four chance nodes represent the chance of rain. The notation indicates that these are the same process, and thus are not independent of each other, and that their probabilities are not conditional on any previous decision nodes. Each branch from a chance node has probabilities as well as descriptions associated with it. These outcomes and their related probabilities may

be the same or different independent of any of the node notation. In addition a further notation has proved necessary when dealing with decision makers who take structural utilities into account. Chance nodes may be connected with a dotted line; this indicates that these nodes are resolved as a single chance event. In this case there will be no probabilities on the branches themselves, the probabilities involved will be those assigned to the relevant chance resolved subtrees. See the figures in section four for examples of this notation.

The purpose of these changes in notation is to increase the availability of information about the random aspects of the tree and still not clutter it over much. This information becomes particularly important when we are dealing with players who find the utility concepts of regret and disappointment significant, as information on the outcome of chance events off the path actually traversed can have an effect on the utility at the tip of the tree actually reached.

Each tip of the tree has a payoff description associated with it. This description has no utility judgment attached to it.

B. JUSTIFICATION OF THE EXTENSIVE FORM

We intend to carry out as much of our analysis as possible in extensive form rather than to collapse the game to strategic or normal form as is often done in Game Theory. Of course the reason that Game Theory does use the

strategic form so often is that that form of the game is much easier to structure for the computations involved with finding a solution to the game. Thus our decision to avoid the strategic form is not without cost. The reasons for it follow.

1. The Heritage of Decision Analysis

One of the major reasons for maintaining the extensive form is that the majority of the audience which we wish to reach and aid with this methodology is already acquainted with Decision Analysis to some extent. They would feel more familiar and have more confidence in a technique which uses a format similar to one which they are used to.

A second related reason is found in the literature on Decision Analysis. A great deal of benefit received from accomplishing a decision analysis is derived from the act of drawing the tree, without having to go through the formal solution process [Brown, 1970]. One of the major goals of any form of Decision Analysis is to increase the understanding which the decision maker has of the situation with which he is confronted. This is easily done in extensive form and cannot be done as effectively with a strategic form model.

The arguments presented above are adequate to support the retention of the extensive form. They are not however sufficient, by themselves, to support the elimination of the strategic form, at least as a computational device. That is left to the following two sections.

2. Solution Objections

Major objections have been made to the use of strategic form in Game Theory. The first comes from the work of Aumann and Maschler [1972] which relates to the invalidity of solutions calculated in normal form when the first node of the game is a move by chance. You will note that this applies directly to the Harsanyi model which we are using. Harsanyi has proposed a modification of his model based on a suggestion by Reinhard Selten. In this Selten model the chance move determining each player's type is the last move rather than the first. This maintains the validity of the strategic form solution but vastly increases the size of the game, as every type of every player must be carried through to the end. However, it seems to us that this objection applies to all games which contain a chance node before a decision node, as the objection would apply to the subgame which had that node as its root.

A similar and perhaps more damaging result is the fact that perfect equilibria of the strategic form of the game and of the extensive form of the game do not necessarily coincide [van Damme, 1983, pp. 127]. This is related to the fact demonstrated by Rees [1984] that one can move from extensive to strategic form and then back to extensive form, resulting in a game which is different from the original one. The mapping between games in extensive form and games in strategic form is not one to one. All of which brings any solution arrived at in the strategic form into question. When we have gone to the trouble of making our extensive form model as close to the reality of the

situation as possible, what is the point of computing a solution in strategic form when we are going to have to check its validity in extensive form afterward?

A recent article by Abreu and Pearce [1984] raises what may be an even more destructive argument against the strategic form. They consider four axioms proposed by E. Kohlberg as standards which an attractive Game Theory solution concept ought to satisfy. They show that this set of axioms is inconsistent in two separate ways, one of which is germane here. They show in their proposition two that there exists no solution concept which is: based on the normal form (axiom 1), nonempty (axiom 2), and satisfies subgame replacement (axiom 4).

(A4i) is similar in spirit to subgame perfection....(A4ii) is motivated by the notion that if the subgame has only one plausible outcome, the solution of the game should be unchanged if the subgame is replaced by that outcome. (A4) is closely related to subgame truncation and subgame consistency.... [Abreu and Pearce, 1984, p. 172]

Based on this result and the desirability of axioms two and four they end their note with the statement, "We conclude that Proposition 2 casts doubt upon the possibility of designing a satisfactory solution concept which exploits only normal form information." [Abreu and Pearce, 1984, p. 173]

3. Structural Objections

Our contention is that the transformation from extensive to strategic form causes a loss of information about the game. As the mapping from

extensive to strategic games is not one to one, this is obvious. We further contend that the information lost may be necessary to correctly solve the game. Specifically, one of our stated intentions has been to be able to advise even those decision makers who are unable to subscribe to the axioms of Decision Analysis. The information lost includes information on risk of the game and on the structural aspects of the utility function. This information is necessary to those persons who consider these elements in their analysis of the game. The information on the process and structural aspects of utility could be used in a strategic form solution after it had been calculated in extensive form. However, the information on risk is another matter.

For people whose preference for probabilities is non-linear, expected utility is not valid as the sole decision parameter, and expectation is the procedure used to reduce the chance nodes of a decision tree to strategic form. (A decision parameter based on more than the first moment would, we feel, also lose its validity if used to take "expectation" in order to collapse a game tree to strategic form.) This is done, effectively, first by moving and combining all chance nodes so that there is a single node at each tip of the tree and then the outcome at that tip is considered to be the expected value of that node.

The following figures, developed from a previous discussion of Allais' paradox [Raffa, 1968], demonstrate the progressive loss of information caused by such a process. If one considers figure 3 as the true state of the world it

would appear obvious that the, mathematically equivalent, figure 4 presents a totally different picture of the situation. Specifically the apparent chance of receiving zero as a payoff is radically different. Figure 5 is the equivalent of a strategic form representation of the game. There is no evidence of the risk in the game in this representation. A person with non-linear preferences toward risk has been deprived of information necessary for him to make a reasonable choice.

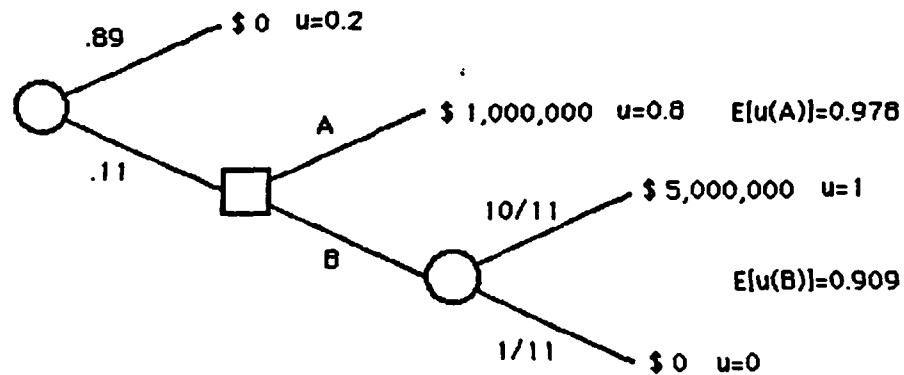


Figure 3
True Situation

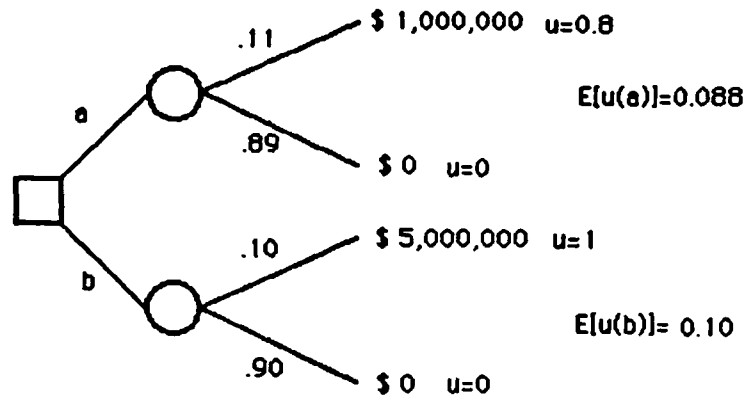


Figure 4
Chance Nodes Moved to Tips

Consider figures 3 and 4 as different situations, which just happen to be mathematically equivalent.

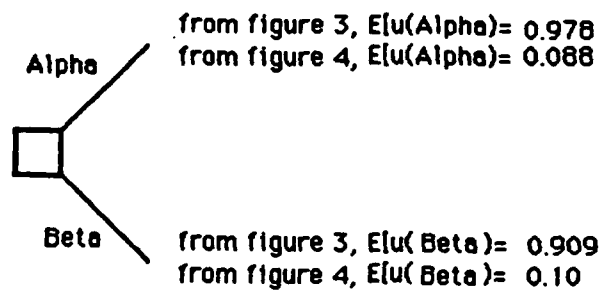


Figure 5
Strategic Form

There is no proper utility function that will allow a person to choose A in figure 3 and b in figure 4, a choice which is perfectly reasonable to a person with a non-linear preference for utility. Thus, as the strategic form of both of these games is figure 5, there is no utility function which will allow such a person to properly analyze these games in strategic form. Obviously any decision arrived at in strategic form will be totally useless to people who have non-linear preferences with regard to probability.

4. Possible Uses of Strategic Form

Due to the drawbacks caused by the strategic form we will be unable to use it as our primary format for the solution of the game tree. However, its advantages for computation can be large and it may be possible to exploit them if it is done with care. There are three possible uses for a strategic form presentation of the game.

The first is in a general solution format. The strategic form could be used to solve games and subgames by direct analysis. Two provisions would have to hold for this to be a reasonable procedure. First, none of the players who have decision nodes within the subgame should be of a type for whom preferences for probabilities enter non-linearly. Second, the proposed solution must be carefully checked against the extensive form of the game in order to ensure that it is in fact a reasonable solution. Clearly this should only be contemplated when the size of the game is so large that there will be significant savings in computation effort by employing the strategic form.

The second possible use of the strategic form is in the modified hypergame analysis of the infinite regress of perception. The current techniques in this area require the use of the strategic form. In the near future, we have little hope of being able to come up with a reasonable presentation of this concept in extensive form. Perhaps with sufficient resort to computer solution techniques and graphics it will be possible to do so at a later date. Until then the same limitations on the use of strategic form must apply to hypergames as to a general solution process.

The third possible use of the strategic form is in sensitivity analysis. Again the same general restrictions must apply. Here it may be possible to make even more use of the computational power of the strategic form as the necessity of repeatedly solving the game greatly increases the computational requirements. This is also one area, when the various strategy options are well known to the persons involved, where presentation in the strategic form may actually enhance rather than degrade understanding of the situation.

C. ASSESSMENT AND MODELING ISSUES

The assessment and modeling of the tree in XDA will be quite similar to the same functions under standard Decision Analysis. The same possibilities for error exist and the same methods for avoiding such mistakes will apply. There are however, three additions to the process which may require new procedures: the use of opponent decision nodes in the model, the increased

capability for, and necessity of, presenting the information available to the various players, and the resultant expansion of the capability of the chance nodes.

1. Decision Nodes

We do not feel that the use of opponent nodes will complicate the modeling of the situation to any significant degree. It seems likely in fact that the ability to explicitly record the presence of decisions made by other players will simplify the process. The major pitfall is one which is also present when one is attempting to model an opponent's decision as a chance node; the determination of what actions it will be possible for the opponent to take. Care must be taken to ensure that all possible actions by the opponent are modeled. Failure to consider a critical option could lead the analysis to provide incorrect advice.

2. Information

There are three major methods in this model of presenting the information which is available to the different players at various points in the game. These are: the information sets associated with the player decision nodes, the notation on the relationship between the chance nodes, and the concept of an information horizon.

The basic functioning of information sets should be familiar to all of our readers. Information sets are used to show the information a player has with

relation to the outcomes of the nodes which are predecessors to that information set, that function is unchanged within this model. However, due to the possible importance in XDA of the results of chance and opponent decision nodes which are not predecessors, a more extensive notation is required in order to record the information state of the player with regard to such nodes. The chance node notation and the information horizon are intended to capture and effectively display this information.

In this context the notation for the chance nodes is used to determine the amount of information available to the players on the outcome of chance nodes which are off the direct path to the root of the tree. There are three levels of such information. The first is that no information is available or this information is irrelevant to the decision maker, i.e., he is not affected by structural utility. Obviously, no special notation is required in this case.

The second level is knowledge of what actually happened at such a node. Specifically, nodes with the same letter designation are deterministically related. That means that having experienced the outcome of one of those nodes a decision maker can determine, with certainty, the outcome of the node which he did not actually experience, even though those two events may have different probabilities. An example would be rolling a single die, one node being whether or not the result was a six, and the other whether the result was even. Unfortunately, not all knowledge of the outcome of chance

nodes is related to experiencing a related node, this is one of the reasons for using information horizons.

The third level of knowledge is knowing what would have happened at a particular node if that path had been taken. This is the information given by experiencing one of a set of nodes which are linked by a dotted line. Chance nodes that are this closely related should not be treated as separate stochastic events, even though they are not precisely the same event. An example would be deciding to play one coin in one of two slot machines, and observing the results of someone else putting a single coin in the machine you decided against. For this reason they are assessed and resolved as a single structure rather than as individual nodes. A detailed discussion of this and related issues appears in the following section.

The subscripts indicate a lesser dependence relation between the nodes. Nodes with the same letter subscript are not independent. Nodes with the same number subscript are conditioned on the same decision or chance node, though they may be independent given that conditioning. These notations are included primarily for the use of the analyst in order to aid his understanding of the stochastic processes involved in the game. For the sake of clarity they may, and perhaps should, be omitted from presentations to the decision maker. Assessment of these factors is covered in section three and specific examples are given in section four.

The purpose of the information horizon is to capture and display the relevant information which is not covered by either of the two methods discussed above. Specifically it concerns knowledge of the outcomes of opponents' decisions or random events which are not predecessors of the decision under consideration. The outcomes of such events could be important due to their effect on the structural utility of tips of the tree. The outcomes of decision nodes would be important for the same reason and also for the information they may reveal concerning the type of the player who made the decision.

A specific horizon is noted by a dotted line; it is associated with a particular decision node and passes through that node and no other. The outcome of all nodes to the left of the horizon are known and the outcome of those to the right are not.

Assessment of the horizons will be a two fold problem. The first will involve obtaining the necessary information and the second the accuracy of that information. The first will simply require asking questions that are not necessary under Decision Analysis. In addition to asking about the nature of the node, we will have to ask when and under what conditions information about the outcome of the node will become available. As with all information one must also ask about the certainty of that information and conduct sensitivity analyses on critical factors where the data is in doubt.

3. Chance Nodes

This section covers modelling and assessing probabilistic situations within the game. We will cover general information and advice. Section four contains several examples of the use of this expanded notation for chance nodes.

The most important information at this point is that modelling situations where random events off the path are significant can become very complicated very quickly. Consequently, the most important advice we can give is that before you make an attempt to model such a situation, you should be very sure that it is necessary. It will only be necessary if one of the players' utility functions includes structural aspects of utility (regret, disappointment, annoyance) and computation of that utility requires information on the outcome of events elsewhere in the tree; i.e. it cannot be adequately modeled using comparison to a prior expectation.

There are two major issues involved in modeling groups of probabilistic events in this context, the actual relationship of the events and the information available on the outcomes of those events. The first can be assessed either directly or indirectly. In the direct method you simply ask; "Is the probability of event A different when decision Z is taken as opposed to decision X?" or "Does the occurrence of A change the chance of B occurring?". In the indirect method you assess the probabilities and use them to check the relationships. This could be done either using traditional techniques or by

presenting the decision maker with chance or fully resolved portions of the tree and asking him to assign probabilities to each realization. We use the term "chance resolved tree" to mean a realization of the game tree where a result is specified at each chance node but all other nodes are left unspecified. A fully resolved tree is where the resultant branch is specified for every node. Assessing probabilities on chance or fully resolved trees is an attractive option leading to easy calculations of the relationship between the nodes. However, it is not clear that decision makers will be able to provide meaningful probabilities using this format as it has not been tested empirically. Unfortunately, certain information states appear to require that probabilities be assessed in this way (see example a below). The best method would probably be to do both direct and indirect analyses of the relationships between the chance nodes, using the assessments to check on the direct questions. The information which one assesses is presented as described above.

The second issue involves the information available on the outcomes of chance nodes off the path to the root of the tree. This can only be assessed by asking direct questions. A major part of the assessment will be discovering whether or not the decision maker will receive information on what would have happened had that branch been taken or merely on what actually happened when that branch was not taken. This also can only be assessed by asking the question more or less directly. This is a result of the fact that this assessment has to do with the perceptions of the decision maker, thus the

Information modeled must be that which exist in the decision maker's perception of reality, not the analyst's. What matters is how the decision maker feels about the information he will perceive. If he feels that he will have information on what would have happened then it must be modeled as such.

4. Examples

In the following simple examples we will attempt to illustrate the issues involved in the modeling of random events given this new notation. All of these examples have the same basic structure. Each is a binary decision, each branch of which is followed by a binary random event. These examples should be viewed as portions of trees rather than whole trees. In such a model two elements must be considered, conditioning and independence.

The first question is whether the probabilities of the random events are conditional on which decision is made. Mathematically when we say that a chance node is conditional on a decision we mean that the probability of a branch of the chance node is different depending on which branch of the decision node is taken. Conversely we say that the chance node is unconditioned if its probabilities are the same for different decisions ($P_a[X] = P_b[X]$).

The second question is whether the random events are independent of each other. We consider a pair of chance nodes to be independent with reference to a particular decision branch if the probability that a specific branch is taken at each chance node, given that that decision branch is taken, is equal to the product of the probabilities of each branch individually, again given

that the specific decision branch is taken ($P_a[X \text{ and } Y] = P_a[X] \times P_a[Y]$).

As conditioning is a binary relation on chance nodes and independence is a binary relation on decision branches this gives us two to the fourth power or sixteen possible examples of the form we have outlined. However, as it does not matter which decision branch or chance node is indicated when one is and the other isn't we only need consider nine examples. We will discuss all three levels of information about off-the-path random events and suggest assessment techniques for each example.

a. Two Dependent and Two Conditional

Consider a decision on whether to hold a picnic in the mountains or at the beach. In either location there is a positive probability that it will rain and ruin the picnic. Further this decision is being made by Attila the Hun and his horde of 10,000 men and if the decision is made to go to the mountains Attila's favorite pastime of burning villages will cause a great deal of smoke which will change the probability of rain both in the mountains and on the beach. At the beach Attila will surf rather than burn villages. Figure 6 is a model of this situation where the structural aspects of the utility function are not important to the player. The outcome of the nodes is not independent and is conditional on the decision node, as the notation indicates, however they can be treated as independent unconditioned events as the outcome of the off-the-path node is irrelevant. The quantiles p and q are merely the

chance of rain at their respective locations given that Attila chooses that location for the picnic.

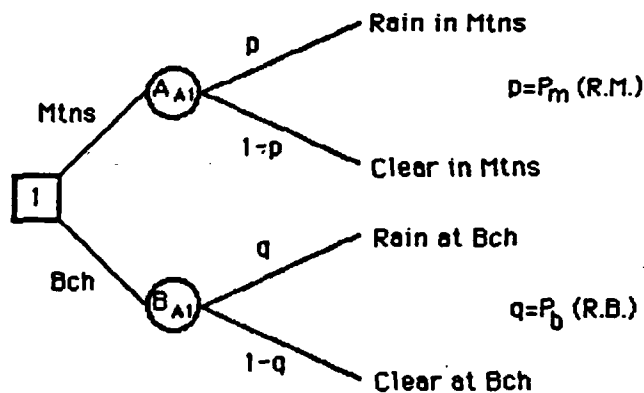


Figure 8
10,000 Man Picnic

Figure 7 represents a situation where the player will gain knowledge of the actual outcome at the off-the-path node and this knowledge is relevant to his utility function. In effect the off-the-path node has been put partially on the path. Note that the notation describes the two nodes as conditioned but independent; this can be done as one node contains all of the information available to the player on the outcome of the decision.

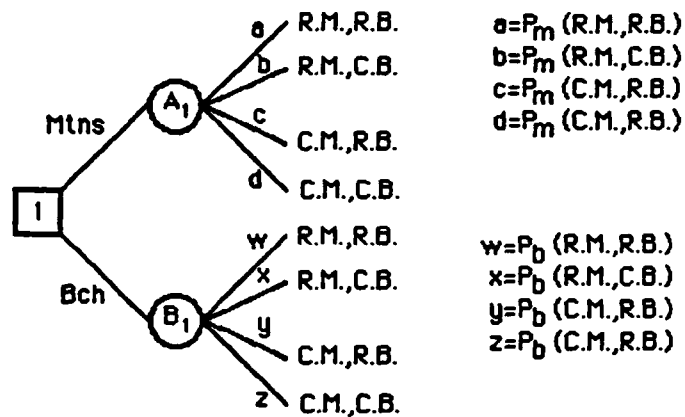


Figure 7
Actual Outcome Model

The quantiles a , b , c , d , w , x , y , and z are conditional probabilities of their respective outcomes given that Attila chooses a specific location for the picnic. Figure 8 shows two alternate forms of the effective result of the chance nodes; either or both could be used to assess the probabilities involved.

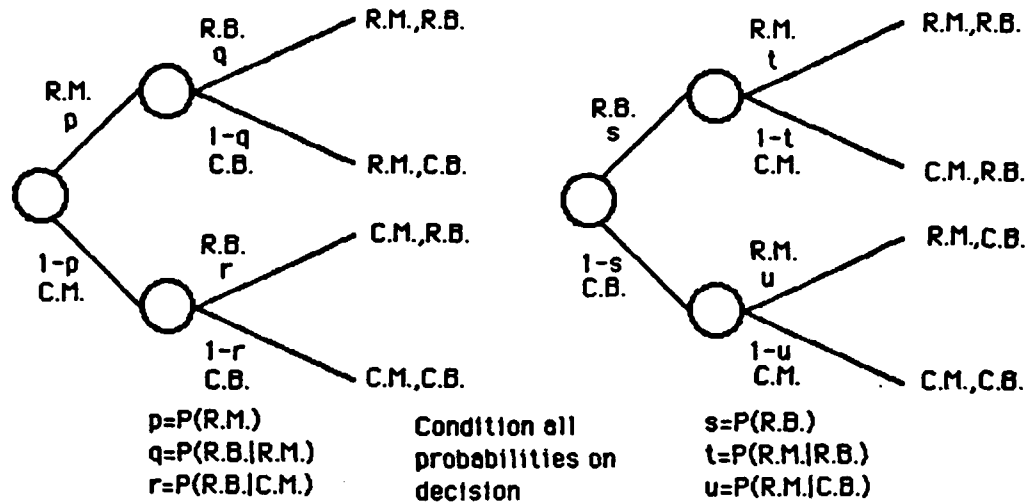


Figure 8
Assessment

Figure 9 is a model of the situation given that the player has information on what the outcome of the off-the-path node would be if the other decision had been made. The nodes are, as indicated by the notation, conditional and dependent; in addition the line connecting them indicates that they must be resolved as a whole. This situation cannot be modeled as in figure 7 because the information available to the player cannot be captured in a single decision node. The reason for this is that in this situation

$P(\text{Rain in the Mountains and Rain at the Beach})$

is based on

$P_{\text{mountains}}(\text{Rain in the Mountains})$ and $P_{\text{beach}}(\text{Rain at the Beach})$

the probabilities are conditioned, but not on the same event and they are not independent. The reverse is not true; the model implied by figure 9 could be used to analyze the situation in figure 7. However, as the procedure outlined in the previous paragraph is more familiar to practitioners of Decision Analysis we feel that it should be used where possible.

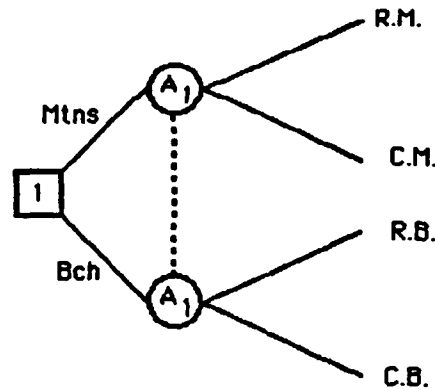


Figure 9
Postulated Outcome Model

For the same reason that figure 9 cannot be represented in a familiar manner, the probabilities associated with it cannot be assessed in the conventional manner. Figure 10 shows the fully resolved trees for which probabilities must be assessed in order to model this situation. These probabilities would be used to directly resolve the coupled nodes dependent on the decision branch taken. The probabilities for trees one through four and five through eight must sum to one.

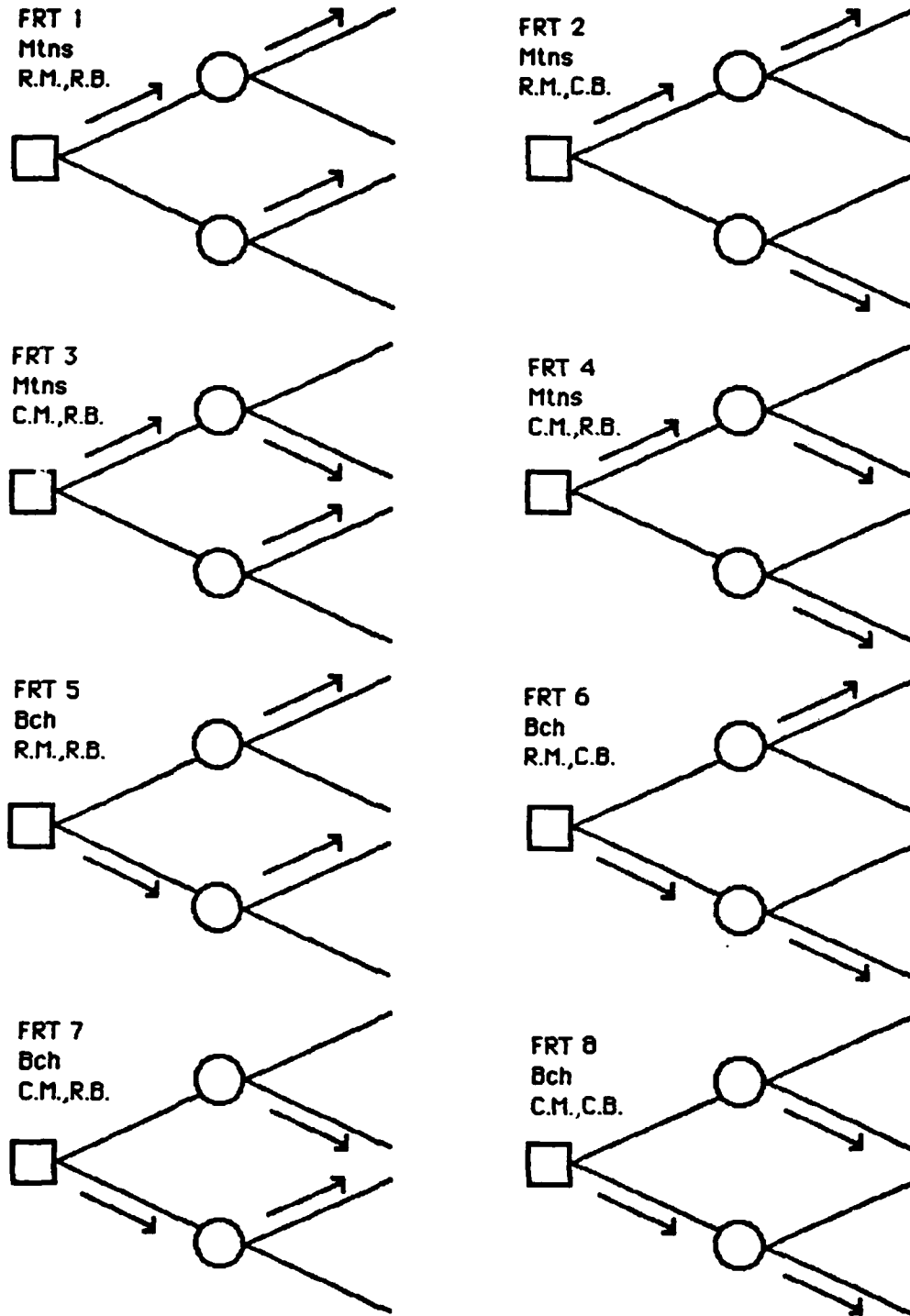


Figure 10
Fully Resolved Trees

b. Two Dependent and One Conditional

Consider an example as above except that Attila only takes 1,000 men along on his picnic. Thus if he chooses to go to the mountains the smoke will only be sufficient to increase the chance of rain in the mountains, not at the beach. The situation where the player has or requires no information on the outcome to the off-the-path chance node is shown in figure 11.

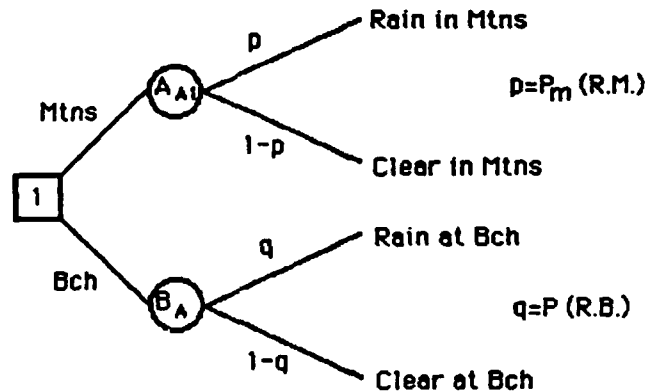


Figure 11
1,000 Man Picnic

The model, given actual information on off-the-path outcomes, is identical to the one in the previous example (figure 7). The only difference is that in assessing the probabilities (figure 8) q , r , and s would be the same regardless of the decision taken. The model for a situation with information available on what would have happened at the off-the-path node is also the same

(figure 9). In this case there is no significant difference in the assessment procedure.

c. Two Dependent and Unconditional

Consider an example as the two above when the decision is made by an ordinary player whose decision will have no effect on the chance of rain. Figure 12 is a model of this situation where the structural aspects of the utility function are not important to the player. Again the outcome of the nodes is not independent but can be treated as such because the outcome of the off-the-path node is irrelevant. The values of p and q are merely the chances of rain at their respective locations.

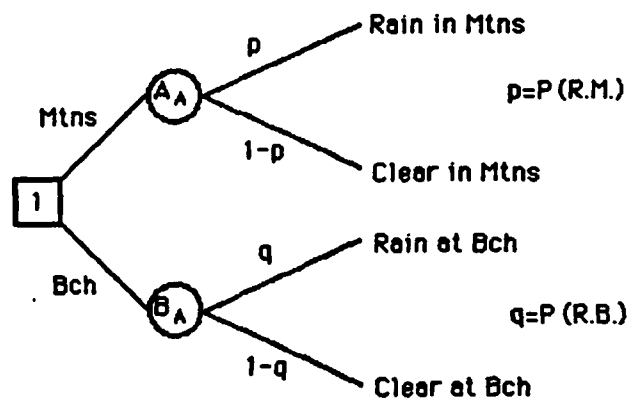


Figure 12
Normal Picnic

In this case as the decision of the player has no effect on the outcome of the random event there is no difference between knowing what actually occurred at the off-the-path chance event and knowing what would have occurred had the other path been taken. Given any information on the off-the-path result the player knows what would have happened. Consequently either method of modeling the situation can be used. For those interested in simplicity figure 13 adequately models the situation. Note that the two nodes are deterministically related, and are in fact identical. Assessment in this case would use figure 8 as the example above, but it is not necessary to condition on the decision branch taken. For those interested in consistency figure 14 is the preferred model of this situation. However, it is no longer necessary to assess the

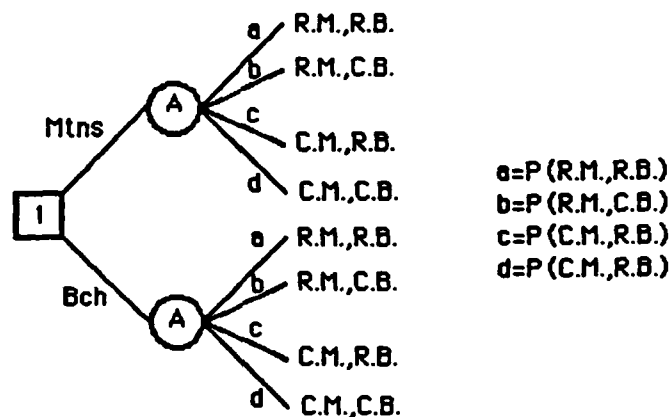


Figure 13
Actual Outcome Model

probabilities on fully resolved trees; chance resolved trees (figure 15) will suffice. If there is some doubt that the chance nodes are indeed unconditional it is possible to assess the probabilities on the fully resolved trees and then check to see if: $P(\text{FRT1})$ equals $P(\text{FRT5})$, $P(\text{FRT2})$ equals $P(\text{FRT6})$, $P(\text{FRT3})$ equals $P(\text{FRT7})$, and $P(\text{FRT4})$ equals $P(\text{FRT8})$.

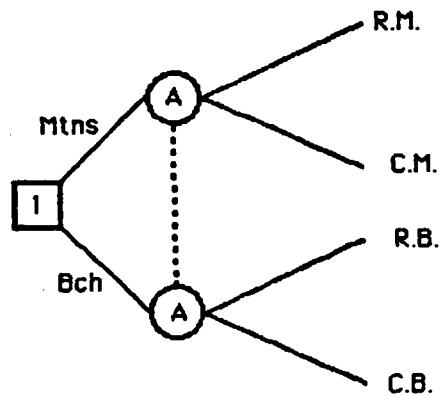


Figure 14
Postulated Outcome Model

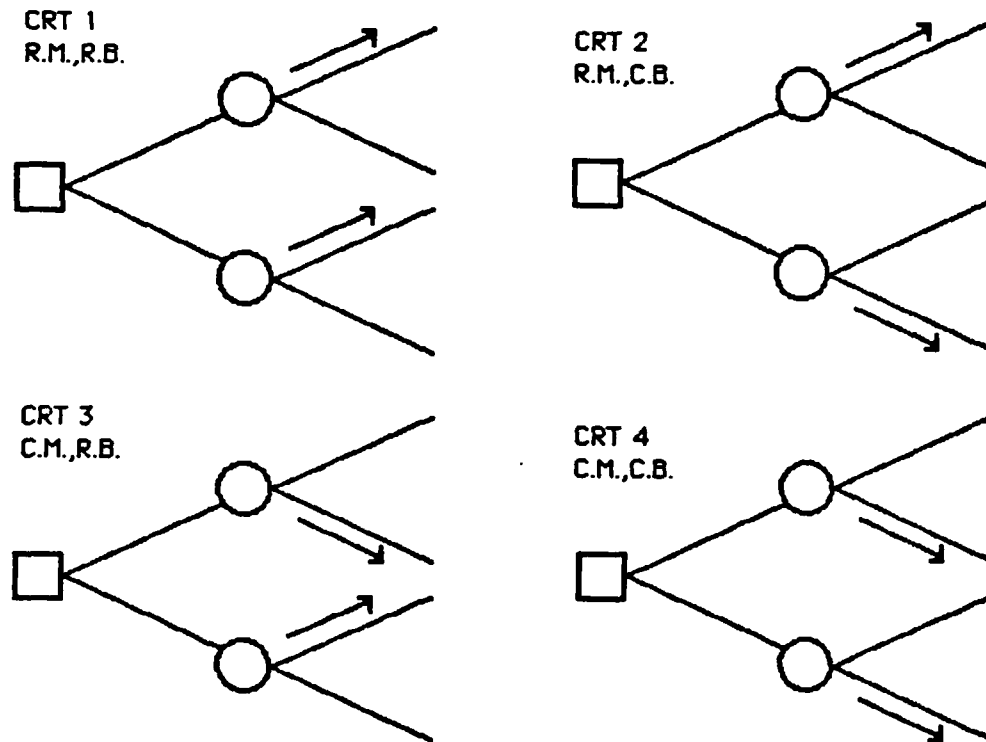


Figure 15
Chance Resolved Trees

d. One Dependent and Two Conditional

Consider the following situation. There are two stacks of cards, 1 and 2, each containing one red and one black card. Chance event A is to randomly turn a member of stack 1 face up; chance event B is the same for stack 2. Decision a is to add one black card to each stack and abide by event A. Decision b is to randomly take a card from stack 1 without looking at it,

place it in stack 2, and then abide by event B. The probability of drawing a black card is $2/3$ if decision a is taken and $1/2$ if decision b is taken, thus both events are conditional. The events are independent if decision a is made and dependent if decision b is taken. Figure 16 shows the model where the off-the-path event is irrelevant. Although the two nodes are not completely independent they may be treated as such.

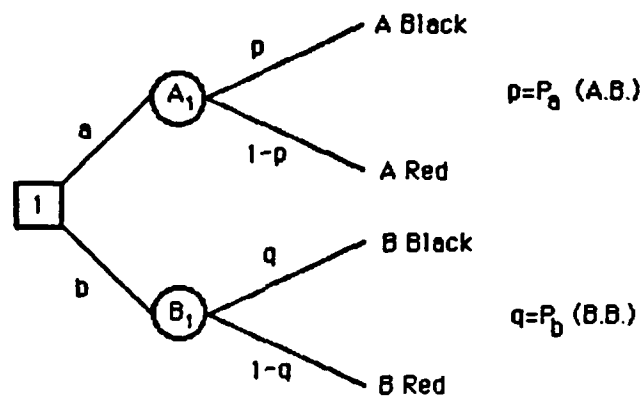


Figure 16
One Dependent and Two Conditional

Figure 17 shows the situation where the player has information on the actual outcome at the off-the-path chance node. Due to the re-modelling the two nodes are now deterministically related and both nodes are conditioned as event A is conditional on the decision made. Assessment would be similar to that used in example a (see figure 19). The probability q will equal r and t

will equal u when conditioning on decision a . The model of the situation where the player has information on what would have happened is shown in figure 18; assessment would have to be on fully resolved trees as in example a.

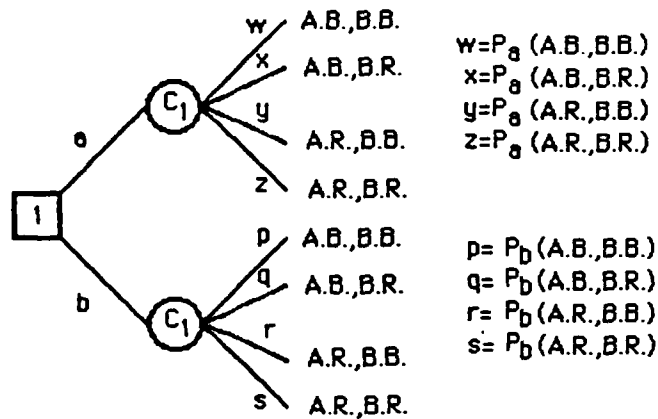


Figure 17
Actual Outcome Model

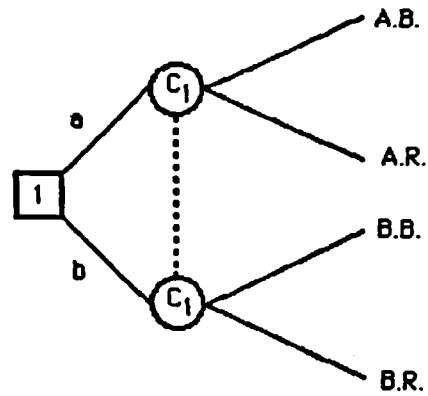


Figure 18
Postulated Outcome Model

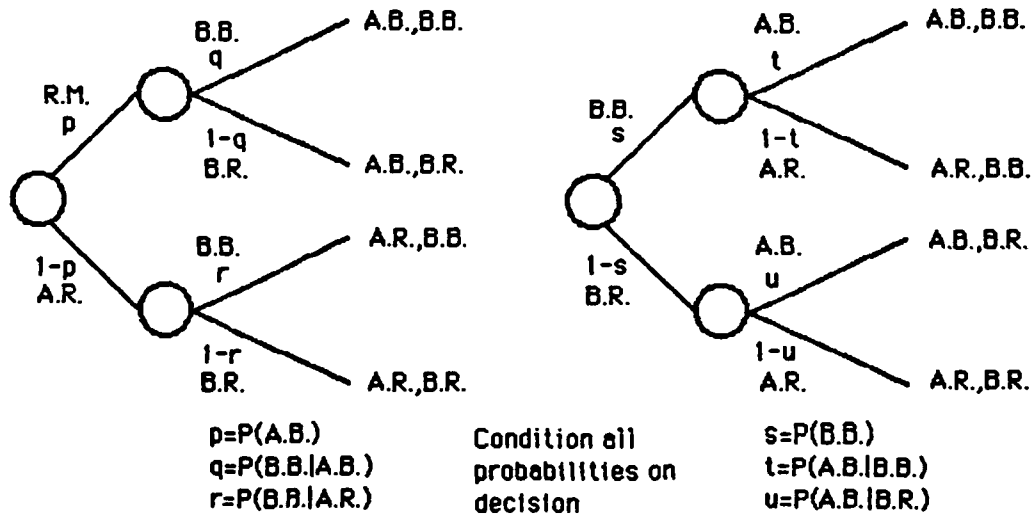


Figure 19
Assessment

e. One Dependent and One Conditional

Consider the situation with the two stacks of cards as above. In this case decision a is to add a black card to stack 1 only and then abide by event A. Decision b is, again, to randomly take a card from stack 1 without looking at it, place it in stack 2, and then abide by event B. The probability of drawing a black card in event A is $2/3$ for decision a and $1/2$ otherwise. The probability of drawing a black card is always $1/2$ for event b. Thus only event A is dependent. The events are independent if decision a is made and dependent if decision b is taken. Figure 20 shows the model where information off the path is irrelevant. Again, although the two nodes are not independent, they may be treated as such.

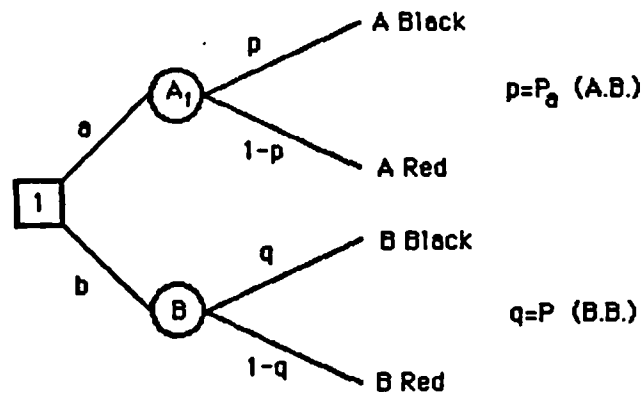


Figure 20
One Dependent and One Conditional

The model of the actual outcome situation will again be figure 17. Assessment, using figure 19, will be easier as s , q , and r will not change when conditioning on the decision and, in addition, q will equal r and t will equal u when conditioning on decision a . Figure 18 will be the accurate model of the situation where the player has information on what would have happened; assessment will need to be made on fully resolved trees.

f. One Dependent and Unconditional

Again consider the two stacks of cards. For this example decision branch a is to simply choose to abide by event A . Decision branch b is to randomly take a card from stack 1 without looking at it, place it in stack 2, and then abide by event B . The probability of drawing a black card for either event for either decision is $1/2$, so the events are not conditioned on the decisions. The events are independent if decision a is made and dependent if decision b is taken. Figure 21 models the situation where information on the off-the-path random event is irrelevant.

As in the third example the decision has no effect on the probabilities of the chance nodes so either the actual outcome model (figure 22) or the model assuming knowledge of what would have happened (figure 23) will be the same. Assessment of the actual outcome model will be somewhat eased as (figure 19) q will equal r and t will equal u and of course we will not have to condition on the decision made. Assessment of the "would have" model could be limited to chance resolved trees.

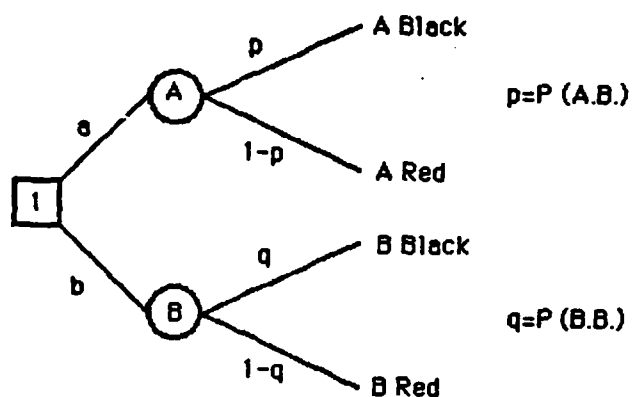


Figure 21
One Dependent and Unconditional

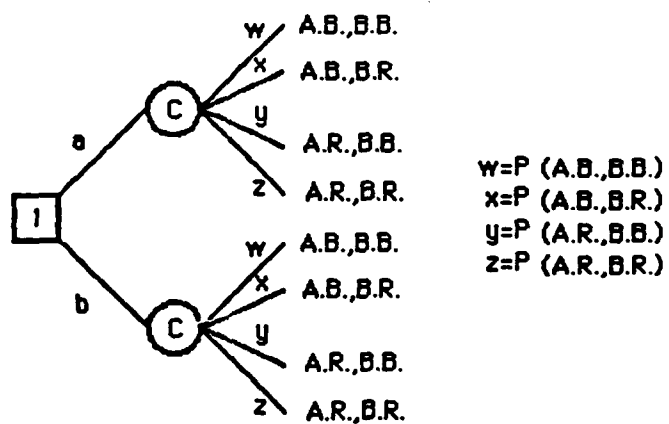


Figure 22
Actual Outcome Model

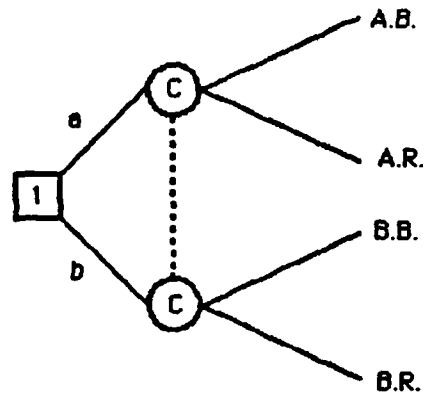


Figure 23
Postulated Outcome Model

g. Independent and Two Conditional

Consider a situation as above except that decision a is to add a black card to stack 1 and abide by event A and decision b is to add a black card to stack 2 and abide by event B. The events are independent for either decision, but their probabilities are conditional on the decision taken. Figure 16 models the situation where information on the off-the-path random event is irrelevant. In this case the nodes are in fact independent.

The situation is similar to the Attila the Hun examples in that the probabilities of the two chance nodes are dependent on the decision taken. The situation where there is information on the actual outcome of the off-the-path node can still be modeled by figure 17. The situation where information is

available on what would have happened had the other decision been made can be modeled by figure 18. Assessment of the actual outcome model would be facilitated in that (figure 19) q will equal r and t will equal u no matter what decision is taken.

h. Independent and One Conditional

Consider a situation as above except that decision b is to abide by event B without adding any cards to stack 2. Thus both events are independent and the probability of only event a is conditional on the decision taken. The modeling of this game is very similar to the one given directly above. The model assuming the irrelevance of off-the-path information is the same as figure 20. The models in the other two situations are identical except that in assessment of the actual outcome model (figure 19) with the additional condition that s is equal to both q and r .

i. Independent and Unconditional

Consider a situation similar to previous card examples; however, decision a is simply to abide by event A and decision b is to abide by event B with no manipulation of the cards in the stacks. Thus the events are independent regardless of the decision taken and their probabilities do not depend on the decision taken. The model for the situation where the information on the off-the-path random event is irrelevant is identical to figure 21.

As the decision chosen has no effect on the probability of the off-the-path event either model of the relevance of the off-path information will be accurate. The actual outcome model will be equivalent to figure 22, with the exception that in assessment one need not condition on the decision chosen. The "would have" model will be equivalent to figure 23, including the ability to do the assessment on chance resolved trees.

Obviously these examples do not cover the whole range of problems that could be encountered in modeling such situations. Issues such as conditioning chance nodes on the results of decision or chance nodes which are not their immediate predecessor, and other even more complicated issues have not even been touched. It is not our intention that this text be a complete manual on how to model competitive situations. Our intention is merely to give the reader a feel for how complicated the situation can become and what kind of issues must be considered.

CHAPTER III

TPOLOGY OF PLAYERS

A. BASIS FUNCTION AND DEFINITION

The concept of a typology of players is inspired by the model proposed by J. C. Harsanyi [1967-8] for dealing with games of incomplete information. We combine this mathematical concept with more general concepts of psychology to identify and make accessible to analysis those factors which govern human action in competitive situations. Our goal is to produce a typology which is large and flexible enough to categorize all reasonable players, and which is still sufficiently well defined that the information is in a usable form.

Mertens and Zamir [1985] have done extensive theoretical work on this subject. They have shown that the concept of type can be rigorously defined

and appears to be internally consistent. They have also shown that the resultant game, which would have an infinite number of possible states as a result of the regress problem, can be approximated by a game with a finite number of states. Our presentation is intended as a practical guide to the application of this concept.

The typology of players has two functions in XDA. The first is to act as a framework for the extraction of the decision maker's knowledge concerning all players, including himself. The second is to enable the application of this knowledge, within the solution concepts of XDA, to reach a "solution" for the game, i.e., to advise our decision maker as to his "best" decision.

Our practical definition of a player's type is the minimum information required to strategy resolve the decision tree for that player. We use the term strategy resolved tree to indicate a tree where all of the decision nodes for a particular player are resolved. It thus represents a particular strategy for him; in this context we mean the strategy that he will use. A choice resolved tree is one which is strategy resolved for all players. Actually it is possible, though not perhaps desirable, to relax this definition and simply require that it include sufficient information, given that the player is of this type and that a particular node is reached, to assign a probability of occurrence to each of the decisions at that node. This is, we feel, too close to the already known Decision Analysis technique of treating the opponent as a chance node and seriously dilutes the interactive nature of the decision making process under

competition. We prefer the original definition. We feel that the majority of the uncertainty about an opponent should be captured as uncertainty about his type.

Thus a type may contain very little or a great deal of information, dependent on the complexity of the game under consideration. However, to adequately fulfill its function in XDA the typology must be able to handle every game. It must also be able to accept any type of valid information from the decision maker. A list of strategy resolved trees would be an effective typology but it would be difficult to assess the probability of the player being of a particular type. We are working toward a language that will be able to describe the whole world, with the knowledge that the decision maker and the analyst will use only that small amount of it necessary to describe their particular situation.

Thus our definition of typology is: a framework for handling and applying sufficient information to strategy resolve any practical decision tree for a particular player. Alternately, it is a method of describing every factor not part of the structure of the game which has an appreciable influence on a player's decisions.

B. OUTLINE

We have identified four major factors (components) which affect a person's preferences and capabilities and thus his decisions, and hence define

that person's type. We describe each briefly in this section and then discuss each in depth in the following sections. Note that the decision maker we advise will be asked for his assessment of each player on each of these four factors.

1. Type of Rationality

This component is the method the player uses to arrive at a decision, the style of decision making. Examples would be Minimax, Nash Equilibrium, "Do the same thing we've always done," etc. Note that with XDA we are attempting to prescribe our decision maker's rationality for one particular decision.

2. Super Utility Function

This component is a description of how the player decides the value to himself of the outcome of a game. It is an important part of the decision process as many types of rationality use this as the major decision criterion. The term "super" could perhaps be replaced with "combined;" the intent is that this component includes more information than standard utility or value measurements.

3. Game Perception

This component is the record of the game that the player thinks he is playing. It includes both the structure of the game as expressed in the tree and the player's assessment of the capabilities of both himself and all other

players. This is intimately tied in with the decision making process, since it is the model on which the player is basing his decision.

4. Type Perception

This component is the player's perception of his opponents' types. It captures the infinite regression of perception and forms the basis for an analysis of the regression of expectations, that is, of the opponents' probable moves.

C. TYPE OF RATIONALITY

The central element of information that we are attempting to capture here is the manner in which a player makes decisions. In fact we are interested, in the information eliciting stage, in the way a particular player will make a particular decision or set of decisions. The framework which we provide for this component must be sufficiently flexible to accept very specific information and sufficiently general to accept very vague information (this can be said of the whole typology). Since, with XDA we are attempting to prescribe our decision maker's type of rationality for the decision under analysis, only the opponents' rationalities must be determined. There are several sources for descriptions of different types of rationality.

1. Basis

March [1978] presented a catalog of alternate types of rationality. Under calculated rationality he included limited rationality, contextual

rationality, game rationality, and process rationality. Calculated rationality is what we in our culture tend to think of as "rationality" (period). Limited rationality includes heuristic search rules, limited calculative ability, and incremental decision making. Contextual rationality models the effect of context as opposed to strictly logical decision variables. Game rationality is the attempt to maximize utility espoused by Decision Analysis and Game Theory. Process rationality covers the effect of the process of the decision making rather than the final decision taken.

In addition March proposes a category called systemic rationalities including adaptive rationality, selected rationality, and posterior rationality. These concepts have little or nothing to do with the standard forms of calculated rationality. Adaptive rationality is the evolution of decisions; those which work are repeated, those which do not are not. Selective rationality is the evolution of decision makers; those who make good decisions survive, the others perish. Posterior rationality encompasses the concept of post hoc rationalization of a decision.

One can argue that these types of rationality can all be modeled as a form of game rationality, maximizing an objective function subject to costs and constraints. This is certainly true of all of the calculated rationalities and may be true of the systemic rationalities. However, we feel that this approach fails to model the actual process which is occurring and thus does not significantly contribute to one's understanding of the true situation.

March's catalog shows that there is more than one way in which rational people think. Since one of our goals is to be able to describe all players which exist in the real world, we must be able to deal with rationalities of any type and with any form of systematic irrationality which we observe in our decision maker's opponents.

2. Possible Descriptions

We have no definitive answer on how one should determine what type of rationality a person is using. Several authors have attempted to devise a method for making this determination; we will discuss some of them in this section.

Goffman [1989], in a work on strategic interaction, proposed the concepts of operational code, style of play, resolve, integrity, and gameworthiness as indicators of how people think in strategic situations. These concepts constitute a series of indicators, or questions which you can ask yourself in order to increase your understanding of your opponent's decision making. For our purposes this is a list of the rough categories of the types of rationalities of opponents, although we include some aspects of operational code under utility.

Porter [1980, ch. 3] gave a similar framework for strategic situations in business. His framework is even closer than Goffman's to a workable typology of players. Porter proposes four main categories in what he calls "competitor analysis:" future goals, assumptions, current strategy, and capabilities.

Current strategy is obviously a key to the rationality of the competitor, as are the assumptions the competitor makes about himself. While we include most of what he calls future goals in utility this area does include information on the structure of the management in order to determine how the goals are arrived at and how likely they are to be held to once enunciated.

The psychological concept of cognitive style, which is receiving a great deal of attention [Goldstein and Blackman, 1978] [Witkin, 1978], also holds promise as a way to help determine what a player's decision will be in a given situation. The research seems to indicate that there is a detectable and consistent pattern in the way a person thinks which allows his preferences and decisions to be predicted to a certain extent. Similar techniques are already in use in the area of personnel management and counseling [McCaulley, 1983]. It is a great leap from this relatively simple application to predicting the actions of a competitor, but given sufficient information about the opposing player it should be possible to use these techniques to aid in determining his probable action.

3. Game theory

Another, and possibly the major, source of descriptions of types of rationality is game theory. The form of rationality generally used in game theory derives from the same roots as the calculated rationality of Decision Analysis, von Neumann and Morgenstern's axioms and Savage's axioms. Game Theory proposes three major types of rationality, which game theorists

call solution concepts. The first is Minimax, which involves selecting the strategy which minimizes one's maximum loss; calculation of one's Minimax strategy requires no knowledge of the opponents' utility functions. The second is Sophisticated Equilibrium (Moulin [1982]), which involves the successive elimination of dominated strategies and requires accurate knowledge of all players' utility functions. The third and most pervasive concept is that of Nash Equilibrium, and its subsequent refinements. This rationality implies an ability and desire to reconstruct any opponent's thought process and a desire to maximize expected utility. It requires a certain amount of information concerning an opponent's utility function, or at least his past performance in equivalent situations.

A fourth form of rationality recently proposed for game theory involves rationalizable strategies [Bernheim, 1984][Pearce, 1984]. This concept shows great promise for use with XDA. They return to the basics, Savage's axioms, and say "this is rationality;" they proceed to develop a concept not based on any of the three solution concepts discussed above.

We note, however, that what is rationalizable for a player depends on his type of rationality, which may be a type of rationality other than that codified by Savage's axioms (or any equivalent set of axioms, such as those proposed by von Neumann and Morgenstern or Pratt, Raiffa, and Schlaifer). Since these alternate rationalities are not as well defined, the solutions based on them will not have the rigor associated with Bernheim and Pearce's

rationalizable strategies. But we must realize that the rationality proposed by these axioms is merely a subset of calculated rationality, which is in itself a subset of rationality in general.

Game Theory makes two assumptions which we reject: that all players are identical except in their utility functions and that all players have the ability and inclination to duplicate any chain of reasoning constructed by any other player. We are working in a situation of inherently incomplete information in which we seek to use to advantage whatever information our decision maker has about his opponents. If we know that a particular opponent, a large bureaucracy for example, practices incremental decision making, we can more accurately predict its actions than if we assume that it uses a less limited form of calculated rationality. Eliciting and applying this valuable information is the function of the rationality component.

D. SUPER UTILITY FUNCTION

The purpose in assessing this component is to determine the factors within the physical outcome of the game which a particular player cares about, including the intensity of his feelings about each factor and the tradeoffs between them, so that we can predict his feelings about possible outcomes.

The first question is what are the underlying bases of these decision variables. Does the decision maker wish to make his decisions in accordance

with the five axioms of Decision Analysis? If not, which are not acceptable? Substitutability? Transitivity? Is expected monetary value a valid decision variable in this case? Must we derive a utility function? Is any type of expectation valid in this case, or should we present the whole distribution (i.e., does the expected value with or without a utility function adequately capture the decision maker's feelings about risk [Lopes, 1984])? Can we derive a cardinal utility function or are we limited to ordinal preferences? These questions and a great many related ones are particularly important when we are attempting to determine the decision maker's type.

After the basics have been established the super utility function itself must be derived. It is possible that what we will be deriving is not in fact a utility function, but merely some decision variable on which the player bases this particular decision. The amount of detail in this function will depend on the complexity of the problem and the amount of information available on this particular player. It could range from a full multi-attribute utility function to a simple preference ordering. We will attempt to define a Super-Utility function which is sufficiently general to contain most reasonable decision parameters within itself. This Super-Utility function breaks into several interrelating factors, some of which may not be applicable to all players.

The first factor is a conventional utility function on the player's own payoff; this is the outcome-based substitutable utility familiar to all practitioners of Decision Analysis. It may be a multi-attribute function including as

possible attributes all costs (process costs, control costs, anxiety, etc.) associated with the path used to reach that end point. It does not include costs associated with branches not traveled. This is more general than utility on the tips of a decision tree; the utility of an outcome may not be separable from the context of the tree. It may be possible to construct a function separable from the context of the tree using ideas similar to Pope's [1984], discussed in Chapter I.

The second factor is the decision maker's utility for the structural aspects of the situation. We call this factor "structural utility" because we feel that none of these factors, however measured, can be represented in a utility function defined simply on outcomes but must be expressed as a utility for whole trees.

Structural utility includes post-hoc outcome analysis such as: regret [Bell, 1982], disappointment [Bell, 1984], and annoyance. Each of these constructs is meant to capture the utility felt by the player as a result of the favorable/unfavorable outcome of a node in the tree. Regret involves player decision nodes, disappointment involves chance nodes, and annoyance involves opponent decision nodes. Each results from analyses of the type, "If only (I) (it) (he) had...." In his paper on regret Bell limits himself to simple trees and requires resolution of all chance events, even those not actually traversed. In his paper on disappointment Bell postulates that a player measures the outcome, not against possible alternate outcomes, but against his expectations.

These expectations are psychological, not mathematical. The manner in which they are arrived at and the actual values depend on the player. It seems reasonable that all three forms of structural utility could be calculated in either manner, via comparisons against actual alternate outcomes, or against expectations, or by some hybrid method such as Kahneman and Tversky's simulation heuristic.

The third factor is utility on the payoffs of other players, both prior to the start of the game (predisposition or prejudice) and at intermediate moves (reward or revenge). This factor could be easily included in a multi-attribute utility function but this might be more appropriately modeled as a changing utility function.

The fourth and final factor is an estimate of how all three of these utilities will change over time (the second guess proposed by March [1978]). This factor is generally ignored by decision makers. This results in more instances of dissatisfaction with the results of decisions than is necessary. Of course the change in utility functions over time cannot yet, and may never, be accurately predicted, but it should be taken into account to the extent possible. There has been some interesting work done in this area by DeGroot [1983] and Cohen and Axelrod [1982] but neither addresses the problem directly. We are investigating the possibility of using a model similar to the one proposed by Brown [1978], which is discussed in detail in Chapter IV, to model the change in utility functions.

Not all decision makers will wish to take all four of these factors into account, at least not to the same extent; not all of their opponents will be affected by them. But this typology is general enough that it will be able to describe the decision parameters of most possible players.

E. GAME PERCEPTION

This component contains all of the information the player has about the game and the capabilities of his opponents. In game theory this information is called the rules of the game. However, our game is not well defined. In some ways the whole world is the game, because we cannot be completely sure which parts of reality are relevant to the game and which are not. Furthermore, we cannot assume that all the players perceive the same game. We have divided the "game," in this sense, into two major parts: the structure of the conflict and measures of power.

The structure is that part of the game which is represented as a game tree. It includes the moves available at each node, the probabilities on the chance nodes, and the outcomes (a description, not the utility assigned to it) at the tips of the tree. It also includes a measure of the information available to each player at each decision node, an information set and/or an information horizon. It is obvious that the perception of this information will differ from player to player and thus several game trees may be needed to describe this structure.

Measures of power include resources, the ability to control the game (size, the relative effect on the market), knowledge, and fallibility (resolve, information state, and resources [Goffman, 1969]). Porter [1980] calls this category "capabilities" and uses it as a general listing of competitor strengths and weaknesses. It could also include, in time constrained games, a measure of speed of decision making and decision execution. In short the measures of power describe the game in an unstructured manner with a special emphasis on those elements not represented in the game tree.

One of the major uses of the two divisions of this component is to cross check one another. Since the perceived structure of the game and the measures of power are closely related we can evaluate important information twice and detect any errors. For example size and resources can have a great deal of effect on which options are available, or which options a particular player thinks are available. Fallibility also has a direct effect on the form of the model. The possibility of errors by a player requires either additional chance nodes or nondeterministic solutions to the opponent decision nodes.

If the structure of the game and the measures of power are so close to each other then why consider them as separate areas? One major reason is assessment. If we just ask our decision maker to draw the tree our opponent would use it will probably look like the one we are using, that is the true tree after all isn't it? But does our opponent know that we will run out of #3 screws if we produce more than 100,000 toasters? What options would we

have if our sales division were four times as large as our research division, like his, instead of vice versa? The tree cannot be drawn until the measures of power are known.

The secret is that there are no game trees "out there." There are only measures of power. The problem is that we can't get solutions from measures of power and vague situational rules; we can solve game trees. Thus the tree is a necessary abstraction, but still only abstraction.

F. TYPE PERCEPTION

The idea which underlies this component is simple; the type of each player includes his perceptions of the types of each of his opponents. Naturally, these player types include their perceptions of the type of each of their opponents, etc. Thus this component is the part of the typology which allows us, as postulated by Harsanyi [1967-8] and formalized by Mertens and Zamir [1985], to capture the infinite regress of perception within a finite typology. In Porter's framework this concept is included under "assumptions," specifically the assumptions the opponent makes about other players.

The major difficulty with this component is the lack of a tested method for assessing this type of information. However, this information on opponent perceptions and the ability to properly analyze it is one of the great potential advantages of the XDA solution concept. The advantage is conditional on the accuracy of the information.

G. ASSESSMENT

Assessment of the types of players will be a major problem as no proven techniques exist. The problem reduces to assessing the type of our decision maker, as his type includes his perceptions of the other players. This helps us to bear in mind that the only information that is available to us is the decision maker's perceptions.

Rationality is no problem at all as the point of the analysis is to prescribe the decision maker's type of rationality for the decision in question. The other components will require more effort.

1. Super-Utility

The procedures to elicit the decision maker's utility function will be a slight variation of the procedures used in Decision Analysis. As in Decision Analysis it is necessary for the analyst to be aware of the possibility that the decision maker is not being totally forthcoming during utility assessment. The analyst can point out to the decision maker the possibility of an increase in effectiveness by deceiving the opponents about the decision maker's true utility function, but that the analyst must know what the true function is to calculate the appropriate deception.

The first major difference will be the very first step. With XDA we realize that there is more than one set of assumptions for the utility function. The first step is to discover which assumptions are valid for our decision

maker, and indeed if the problem under consideration warrants the construction of a utility function at all. The spectrum of possible functions would range from a classical, substitutable, univariate utility function to a time variable, non-substitutable, multi-attribute utility function over strategy resolved trees including process and structural utilities to a simple probability distribution over physical outcomes. We must explain to him the effects of the various possible combinations and aid him in determining which of the possible decision variables is valid for him.

Once the basis of the function has been determined, the methods used in Decision Analysis should prove a sound basis for this assessment, with certain changes. For example, a consistent method should be developed to predict or deal with time variable utility functions. In addition the effect of modified utility theories, which discard certain axioms [Machina, 1983] [Munera and de Neufville, 1983], on assessment issues must well understood and documented. If the developers of these theories are working on such issues, then the main task for XDA is to fit this knowledge into our framework.

2. Game Perception

The majority of the work in assessing the decision maker's perception of the game is done in the construction of the game tree. The differences between standard Decision Analysis and XDA in that process have been covered in chapter II. However, one should remember that this is a competitive situation; an unexpected or unorthodox move may give an advantage. A

thorough assessment of the decision maker's measures of power accompanied by a brain storming session should discover such "hidden" moves. In addition it would check the tree portion of the game model.

3. Type Perception

The real assessment problem involves deriving the decision maker's perception of his opponents' types, including their perception of their opponents' types, for this forces us to directly confront the infinite regress of perception. Naturally this is the area of knowledge which holds out the best promise for a significant improvement in competitive decision making, a systematic method for putting yourself in your opponent's shoes. There are several possible problems with the assessment of this component.

First, much of the data required may be unavailable or obtainable but costly. Costs must be figured in the model, increasing its complexity. Even if the decision maker had heeded the advice of Porter [1980] and maintained a comprehensive intelligence dossier on each opponent, this data must still be processed to fit the typology of XDA. Gathering as well as applying this information could become a major function of the analyst.

Another problem stems from the facts that we are dealing with an infinite regress of perception and that our reservoir of information is finite. It will be necessary for the analyst and the decision maker to ensure that the assessment of this factor is based on reasonable data and subjective feelings and does not degenerate into idle and profitless speculation.

There will also be the problems of misinterpreting data and succumbing to wishful thinking, such as underestimating the moves which an opponent will see, or believing that the opponent uses an inferior rationality in his decision making process. This tendency to underestimate the opponent is less pronounced when working with a well defined methodology such as XDA; this is one of the few places where such biases can get into the analysis.

It is probable that the decision maker will not be able to determine an opponent's type with certainty. The decision maker must then determine his subjective distribution over the possible types for that player. If the decision maker can only narrow down a particular opponent to three types of rationality and four forms of super-utility function it would be better to obtain the conditional subjective probabilities of each component and calculate the probabilities of the 12 possible types rather than attempt to assess them directly.

CHAPTER IV

SOLUTION METHODS

In order to "solve" a game tree decisions must be specified at the opponents' decision nodes and the decision maker's decision nodes. Our current thinking is that this analysis should provide $P(\text{choice } i | \text{player is type } j \text{ and we are at decision node } k)$ for all k , decision nodes in the tree, for all i , branches of decision node k , and for all j , the possible types of the player whose node k is. There are thus two possible ways to model uncertainty about the behavior of a player: with these conditional probabilities or with the decision maker's probability distribution on the type of each of his opponents.

We feel that it is better to model the majority of the uncertainty about the behavior of the player, either in general or at a particular node, as uncertainty about his type. This means that our solution method should deliver a

deterministic result at each node, that is, the conditional probabilities of all choices save one should be zero or close to zero. By "close to one" we do not mean a concept of small ϵ probabilities, rather significant and assessable beliefs on the part of the decision maker. There will be situations where it is not possible to model all of the uncertainty in the distribution over types. Our tree solution concepts can still be applied to such games.

We intend to use the concept of rationalizable solutions [Bernheim, 1984] [Pearce, 1984], as the basis for our method of solving individual decision nodes. We will cover this subject in more detail in Section B of this chapter. An obvious exception to this use of rationalizable strategies is the decision maker's decision nodes, where we intend to prescribe the branch chosen. One major issue to be addressed in such prescription is whether we are to consider pure choices and strategies only or must we consider random "mixed" strategies as well? This is the issue which is addressed in the following section.

A. RATIONALE FOR NOT USING MIXED STRATEGIES

Traditional Game Theory includes the possibility of mixed strategies and randomized behavioral strategies. Indeed, in many games the Nash equilibria require the use of such strategies. However, many of the people interested, as we are, in applying game theory concepts to real world situations have not used mixed strategies [Fraser and Hipel, 1984] [Bennett, Huxham, and Dando, 1981]. We agree with this approach for two major reasons.

The first is the practical issue of what one can get a decision maker to accept. The second is the issue of whether it is, in a Game Theoretic context, necessary to consider mixed strategies in games of incomplete information.

1. Practical Objections

The major practical difficulty has to do with the perceptions of the large majority of decision makers. In general, business decision makers like to think that they are in control of the situation, that they can understand and analyze the situation well enough to foresee every eventuality and thus control or at least predict the outcome. It has been difficult for analysts to get decision makers to accept the probabilistic modeling required in standard Decision Analysis [Brown, 1970, p. 86] [Hogarth, 1975, p. 273]. Carrying this further, one can imagine the difficulties involved in convincing the Board of Directors of a large corporation to allow the decision made on a multi-million dollar project to be determined by a random event.

In attempting to argue for mixed strategies one finds that the usual explanation for why such a randomization may be necessary, that one needs to randomize in order to hide your intention from the opponent, falls in a practical situation. There are many ways to hide your intentions from your opponent in the real world which do not involve using a random process. Secrecy and deception fulfill the same goal, randomizing your opponent's information rather than your decision process. In addition in many situations merely randomizing the decision will not guarantee that the decision will not

be discovered by an opponent. Any sufficiently complex decision must be coordinated with peers and transmitted to subordinates and is vulnerable to discovery by an opponent during this process.

2. Theoretical Objections

The majority of the theoretical objections to the necessity of randomized strategies seems to rest on this last practical point. The existence of incomplete information seems to cause enough uncertainty for your opponent that you do not have to introduce more.

John Harsanyi [1982], in an attempt to demonstrate the stability of mixed strategy equilibria, examined games with randomly disturbed payoff vectors. These games were constructed in such a way that only the particular player knows his exact payoff, the other players only know his "base" payoff and the distribution of a random variable which modifies it. This is obviously a situation of incomplete information. Harsanyi argued that mixed strategy equilibria in standard games were generally stable as they could "almost always" be obtained as a limit of pure strategy equilibria in a disturbed game. In the course of this argument his Theorem 2 shows that every equilibrium of such a disturbed game is a pure strategy equilibrium. In addition he says:

Theorem 2 and the Corollary to Theorem 1 can be regarded as extensions of the results obtained by Bellman and Blackwell [1949] and by Bellman [1952]: If the game itself already contains enough "randomness" (random variables) then the players themselves need not — indeed, should not — introduce any additional randomness by a use of mixed strategies." [Harsanyi, 1982, p. 82]

Aumann, et al., [1981] attempt to make this concept more precise by determining exactly what conditions are necessary to allow mixed strategies to be replaced by pure strategies. Specifically they consider games in which each of n players takes an observation of the nature of the game (his prior information). Their results are as follows. If players 2 through n by pooling their information cannot ascribe positive probability to any specific observation by player 1 then player 1's mixed strategies may be replaced by pure ones. And if no player can, from his own observation, ascribe a positive probability to a particular observation of another player, mixed strategies cannot, in general, be purified but every Nash equilibrium in mixed strategies can be replaced by an approximate equilibrium in pure strategies which yields approximately the same payoff. Our intuition of what these restrictions entail in a real world situation is that in each case the players are reasoning based on a probability distribution which is roughly analogous to a probability density function over a segment of the real line. This does not seem to be an unreasonable restriction. Even if the number of states of the definition is finite, but large, and the probabilities assigned to each are greater than zero, but small, it seems reasonable that these results would still hold, in an approximate sense.

The second question on this issue is whether in restricting ourselves to pure strategies we have set up a situation where we will not have a reasonable solution for every game, especially as it is well known that not all games have

Nash equilibria in pure strategies. Our decision to use rationalizable strategies rather than equilibria as our main solution concept comes to our aid. We are guaranteed that every player has at least one pure rationalizable strategy. Pearce [1984, p. 1034] states this explicitly and Bernheim [1984, p. 1016] implies it.

From this discussion it should be clear that it is reasonable to restrict the solutions we consider to the set of pure strategies. However, these arguments for the exclusion of mixed strategies are only to support our feeling that they are not necessary. We do not intend to imply that it has already been proven that mixed strategies are unnecessary; that is for the future, if ever. Note that our model as formulated can handle mixed strategies on the part of the opponents although our bias toward pure strategies is shown in our intent that the probabilities of all but one choice should be close to zero for each particular type of the opponent. Significant changes would have to be incorporated into the solution concept in order to support the selection of mixed strategies by the decision maker.

B. OPPONENT DECISION NODES

We have selected a highly modified form of rationalizable strategies as the heart of our method of solving opponent decision nodes. Roughly, a rationalizable strategy is one about which the player can construct a believable and consistent story which will lead to its selection. As we mentioned in

Chapter I what constitutes a rationalizable strategy will certainly depend on the type of rationality of the player. This applies both to the player that we are advising, if we are using the model of player 2 as "Future Decision Maker" (see Chapter V), and to his opponents.

As we intend that the majority of the uncertainty associated with the decision nodes be modeled in a probability distribution over opponent player types, the choice of a branch at a particular decision node should be nearly deterministic given the type of the player. Any residual probability on the non-primary choice would be considered a model of the chance of error on his part, information which he may have that is unknown to us, or other unforeseen circumstances. We prefer this model because it allows us to extract more information on an opponent's subsequent moves by examining the moves he has already made.

However, the set of rationalizable strategies is large, and in the broadest sense all strategies are rationalizable. Obviously it will be necessary to restrict the available set of rationalizable strategies. We will use the type of a player and the concept of rationalizable strategies to narrow the set until there is a single member. Rationality obviously reduces the set; reasonableness limits the set of strategies to those which are not strictly dominated, cautiousness limits it further, etc. Game perception also has an effect in that a player will not choose an option he does not see, and will be more likely to choose an option whose counter move he does not see. Type perception also

plays a role as a player will certainly consider his opponents' actions when selecting a course of action. The super-utility function will control which options, in this restricted set, the player prefers.

The implication is that if we cannot make a decision deterministic or "nearly deterministic" we return to the original distribution over player types and sub-divide it until all of the subsequent decisions are so. However, if there is insufficient information to do this or if the decision maker or analyst is uncomfortable in doing so the whole game solution concepts will accept significant deviation and will still produce valid answers. The only result is that the Bayesian updating at that particular node will not provide as much information on the player type as for one which is nearly deterministic.

You will note that we have omitted any specific method for the solution of opponent decision nodes. The reason for this is that we do not yet have any hard and fast answers for this very complicated situation. We intend, obviously, that this be a major focus of research in the future. What we have listed here is a set of general guidelines, which will serve to give direction to that research, if nothing else.

C. WHOLE GAME SOLUTION

This section discusses the method of selecting the decision maker's first decision. As has been argued by other authors [Rees, 1984] [Fraser and Rees, 1985] we feel that this is the sole point at which Decision Analysis, and by extension XDA, is prescriptive. We attempt to be more flexible than Decision Analysis in the manner in which this choice is calculated. We strongly believe that this selection must be made in accordance with the decision maker's super-utility function, including his feelings on the validity of the axioms of Decision Analysis.

There can be several techniques for analysis of such a decision based on the decision maker's preferences. This appears to us to be a very profitable area for future research. We have identified two major issues in this context. The first is the effect of the belief or disbelief of the decision maker in the axioms of Decision Analysis on the validity of the computational techniques. The second is the effect of various simplifications which can be made in the solution procedure in order to reduce the computational complexity of the solution.

1. Decision Maker Preferences

We have, at this time, identified two general types of solutions which appear to us to have validity given certain preferences on the decision maker's part. The first, if he accepts the axioms of Decision Analysis, is a relatively simple modification of the familiar "average out and rollback" techniques used

In Decision Analysis. The second is a more general analysis which delivers by analytical or numerical techniques a "risk profile" [Hax and Wllg, 1977] for each of the decision maker's alternatives. This could be a profile either over raw outcomes or over the decision maker's utility function. In the terminology of XDA we would deliver to the decision maker a distribution over the outcomes for each strategy resolved tree.

If the decision maker subscribes to the axioms of Decision Analysis, once the decision nodes of all players other than player 1 have been resolved the situation becomes one familiar to practitioners of Decision Analysis. We either average out and roll back or merely average out if the model used is the one based on the suggestion by Brown [1978]. One major exception to this simplicity is the ability to update the probability distributions at the opponent decision nodes in accordance with Bayes' Rule.

The probability distribution at a diamond node has two elements. One is the distribution over the choice made by that player, given that he is a particular type, which is delivered by the node solution procedure discussed above. The other is the decision maker's subjective distribution over the type of that player. The very fact of being at a particular node within the tree may contain information about what that subjective distribution should be. Thus it should be possible to use Bayes' rule to update this distribution based on that information.

Consider one of an opponent's decision nodes, one which has another of that opponent's nodes as a predecessor. We have analyzed this second node in accordance with the opponent node solution procedure and thus have already determined $P(\text{choice} = j \mid T_s = i)$ for all j , possible choices at the node, and for all i , possible types of the opponent. The vast majority of these quantities should be zero or one. Now we desire $P(T_s = i \mid \text{at node})$ which requires the use of Bayes' rule in the following form.

$$P(T_s = i \mid \text{at node}) = \frac{P(k \mid T_s = i) \times P(T_s = i)}{\sum_j P(k \mid T_s = i) \times P(T_s = i)}$$

We have analyzed the opponent's prior node and have already determined $P(k \mid T_s = i)$ for k , the choice at the previous node which resulted in our eventually reaching this node, and for all i , possible types of that opponent. We also have $P(T_s = i)$ for all i , our prior distribution, and so already have the information required to calculate $P(T_s \mid \text{at node})$. Thus we can calculate the overall probability distribution of the choices at the node:

$$P(\text{choice} = j) = \sum_i P(\text{choice} = j \mid T_s = i) \times P(T_s \mid \text{at node})$$

Note that we do not have to worry about the effect of a player being type m at a node which he could not have gotten to if he was in fact type m . The Bayesian updating would force $P(T = m \mid \text{at node}) = 0$ as $P(k \mid T = m)$ would necessarily equal zero.

In these calculations we have assumed that nothing other than decisions by that opponent affect or reveal information on an opponent's type. If this

were not so, for example a model of a changing utility function, then one would have to update the distribution of the type at every node which could possibly affect it. Also if an opponent's type is such that he has a subjective distribution over the types of his opponents it would be possible to use Bayes' rule to update his distribution in a similar manner. One would have to address the issue of whether he would use correct Bayesian updating or some faulty heuristic; this information would have to be included in his type. Once all of the diamond nodes have been solved and all of the appropriate updating has been done, simply average out and fold back as necessary and select the strategy with the highest expected utility. Note that this will probably have to be an iterative process, if the distribution of the choices of a type of player depends on his subjective distribution. The Bayesian updating will work from root to tip of the tree, with the time sequence of the tree, while the solution of the decision nodes would work best in the manner of dynamic programming, from tip to root.

One of our major goals is to provide a method of conducting a valid Decision Analysis for those people who do not accept one or more of the axioms of Decision Analysis, such as those people for whom expected utility, by itself, is not a valid decision parameter either because of non-linear attitudes about risk or because of the importance of some element of process or structural utility. The following method is similar to that described above in its analysis of the probabilities on the diamond (opponents') decision nodes;

the square decision nodes are treated differently.

Instead of calculating the expected utility and assuming that our decision maker will abide by that decision we follow the lead of Hax and Wilg [1977] and present the decision maker with a distribution over the outcomes. There would be one distribution for each strategy resolved tree. They could be presented as probability densities or cumulative distributions, if the type of outcome allowed accumulation. The distribution could be over either utility, the raw outcomes, or merely which tip of the tree is arrived at. The opportunities for creative and effective presentation of the data are limitless. In every case, however, the distribution would be derived from the probabilities on the chance nodes and the probabilities on the diamond decision nodes.

The decision maker would then determine his preferences by examining these distributions. Some could be eliminated prior to presentation by such techniques as stochastic dominance, if the preferences of the decision maker will support such a procedure.

2. Computation Issues

There are two computational issues which we would like to address. The first is the increase in the complexity of the computations involved in solving the tree if Bayesian updating of priors is carried out. The second is the issue of the computation of the distributions to be delivered when the alternate solution method is used.

It should be obvious that the use of Bayesian updating in the solution process, while a major strength of XDA, will greatly complicate the calculation of the probabilities associated with the diamond nodes. This will especially be the case if one or more players' type is affected by nodes other than their decision nodes; one can imagine a game with a large tree and many players and having to update for each type of each player his subjective distribution of every other player at every node. It may be possible to eliminate some players or types from the process when they do not maintain such a distribution or it does not affect their decisions. It will be a difficult question for the analyst whether this added complexity is justified; unfortunately we have no good advice on how to answer it.

There are four levels of complexity in implementing the Bayesian updating: none, ours only, theirs only, and everyone's. Conducting no updating can be viewed as similar to the Harsanyi model, beginning the game with a large chance node representing the subjective distribution of the decision maker over the types of his opponents, and the effect of any subjective distributions maintained by those opponents being factored in, either implicitly or explicitly, to the choice distributions at the diamond nodes. The second level seems very reasonable in that we are updating only that set of distributions about which we have the most information, the decision maker's. The third level of complexity appears to us to have no great validity; the practical situations would be few and far between where it would be profitable to update your

perception of an opponent's subjective distribution and not update your own distribution. The fourth level is what we discussed in the preceding paragraph.

The second major issue is the computation of the presented distributions in the distributional solution method. We would prefer to derive these distributions analytically. But this may be very difficult to do, especially with very large and complex trees or trees which have continuous distributions at chance nodes. Indeed Hax and Wllg [1977] generated their distributions using a Monte Carlo method. The decision on whether to use analytical techniques or Monte Carlo is one we will again have to leave up to the analyst. Our only advice is two rather obvious observations: first, that the two methods are theoretically equivalent, and second, that if you use Monte Carlo, variance reduction techniques are in order. We think that common random numbers would be one valid technique.

D. SENSITIVITY ANALYSIS

Even more than in Decision Analysis sensitivity analysis in XDA will be a major factor in understanding the situation and making a decision. This is due both to the increase in the complexity of the model in XDA and to the addition of an additional concept of sensitivity analysis.

Standard sensitivity analysis of the solution to a game tree will have more parameters to consider than in traditional Decision Analysis. In

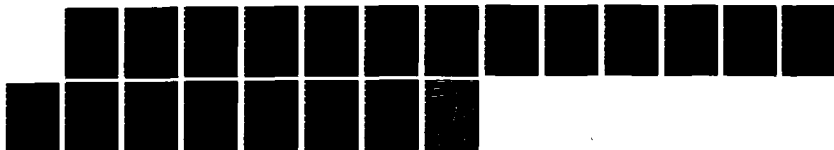
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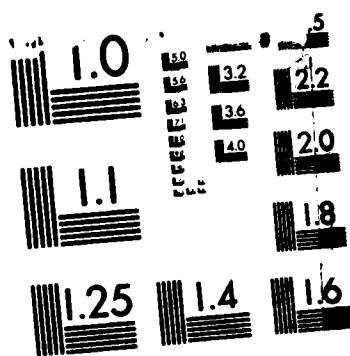
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addition to the probabilities on the chance nodes there are the probabilities on the types of the various opponents and the different probabilities included in the type (e.g. chance of error). In addition to the physical outcomes and associated utility for the decision maker, these aspects must also be considered for each type of each opponent. However, there is no qualitative difference in the way these quantities need to be examined. What is different in XDA is, because of the addition of competitors to the situation, the increased need to do "sensitivity analysis" on elements of the game which are neither outcomes nor probabilities but are the players' perceptions of each other and of the reality of the game.

Part of such a non-quantitative sensitivity analysis can be carried out just by careful and systematic consideration of the game with an emphasis on finding things that may have been left out of the model. XDA already provides an inclination toward this function as we include the components of game perception and type perception in our typology. However, we feel that a more structured approach is necessary. We want to provide the decision maker with a method of dealing directly with the infinite regress of perception and expectation. Fortunately, there are people working in related areas to ours whose ideas show promise for being adapted to our purposes.

One of the great needs of any methodology which intends to advise people in competitive situations is some method of capturing the interaction between the players. Each player is to put himself in his opponent's shoes; of

course the opponent is trying to do the same, which brings about the infinite regress: "He thinks that I think that he thinks...." This applies equally to the player's intentions in the game and to his perceptions of reality. In chapter I we mentioned that other people have been attempting to apply the concepts of game theory to real life situations. Bennett and others [Bennett, 1977] [Bennett, Huxham, and Dando, 1981] [Bennett and Huxham, 1982] have developed the concept of Hypergames for modeling competitive situations where there are misperceptions of the game. Fraser and Hlppel [Fraser and Hlppel, 1984] [Takahashi, Fraser, and Hlppel, 1984] have developed a simplified method for analysis of large games which is applicable to Hypergames as well as regular situations. They have unified these techniques into a single methodology and call it "Conflict Analysis."

Hypergames deal with games in strategic form and only require an ordinal assessment of utility. As the model is in strategic form it assumes that the game is either deterministic or that using expectation does not distort preferences. A first level hypergame is considered to be a representation of the game each player thinks he is playing. Figure 24 is a simple two person hypergame. A second level hypergame of the same situation would include these matrices and the game player 1 thinks player 2 thinks he is playing and the game player 2 thinks player 1 thinks he is playing, that is, the perceived first level hypergame for each player. This sequence can be continued to the extent necessary to model the game correctly. This shows promise as a

method of dealing with the regress problem. In our model we would deal with only the decision maker's perceptions of the game. But this should be a good method for looking at the possibilities of the situation from the other players' perspectives.

<u>Player 1's Game</u>		<u>Player 2's Game</u>	
Player 2		Player 2	
Player 1	1,4 2,3	Player 1	2,4 1,3
	4,2 3,1		4,1 3,2

Figure 24
Simple Two Person Hypergame

Conflict Analysis includes the possibility of modeling the situation as a hypergame. The solution concept proposed is one of various forms of "stability." The basic argument is similar to Nash equilibrium, the unavailability of unilateral improvements. The Nash concept is weakened, or strengthened depending on your viewpoint, so that solutions are always available in pure strategies. The main advantage is that computer algorithms have been developed to solve games which are relatively large.

We feel that Conflict Analysis has at least two of the weaknesses of Game Theory as a whole. First, it is mutually prescriptive to all players.

They are interested in finding the solution to the game instead of helping one player win. Secondly, there is too much reliance on equilibrium style solution concepts for what is really a one shot competition. Hypergames, in addition, use the strategic form, which we argued against in chapter II. Because of the complexity of any hypergame presentation it is unlikely that we can escape the use of the strategic form in this case.

We still have a great deal of work to do in adapting these concepts to fit with the philosophy and methodology of XDA. Many questions must be answered. For example, is our modification of rationalizable strategies adaptable to hypergames? How do you tell how many levels of regression are necessary for a proper analysis? This is a very promising research area. However, we feel that hypergames coupled with a suitable solution concept can be used successfully to give insight to the structure of the game and to provide a simple format to derive and analyze several levels of regress. It will thus provide a valuable addition to XDA's sensitivity analysis arsenal.

CHAPTER V

APPLICATION ISSUES AND SUMMARY

A. OVERVIEW AND APPLICABILITY

We have now looked at all of the pieces of XDA. It is time to see how all of these components fit together. A brief general chronology of a hypothetical Extensive Decision Analysis follows. This description is of a single analysis, however like Decision Analysis, the proper way to conduct such a study is to use an iterative procedure while gradually increasing the complexity of the model.

The analyst and the decision maker model the game in extensive form (as a decision tree). They may decide to represent the decision maker by one or more players. The probabilities associated with the chance nodes are determined in the same manner as Decision Analysis. The decision maker's

type is then determined. This includes his preferences with regard to the axioms of Decision Analysis, his utility function and an assessment of the types of the other players involved. If a single type cannot be specified for a player then we assess the decision maker's subjective probability distribution over the range of possible types. The decision maker's perception of the game perception component of the opponents' types is cross checked against the game tree to see if the model is accurate.

Now the model is complete and we are ready to attempt a solution. The concept of rationalizable strategies is used to determine a specific choice or range of choices at each opponent decision node. Then the desired level of Bayesian updating is used to assign the proper probabilities to each decision node which is not a decision by player 1. Once the opponent nodes have been resolved the tree is solved according to the decision maker's type, either with an expected utility analysis (average out and fold back, or simple average out), or a direct or Monte Carlo analysis delivering an outcome distribution over the set of strategy resolved trees with the decision maker making the final selection. In either case the final result is a value function over strategy resolved trees.

Before the final decision is made we perform sensitivity analysis on the structure and the probabilistic values of the game tree. In this process we use both traditional techniques and Hypergame analysis to gauge the possible effect of misperceptions of the structure of the game, perhaps going so far as

to advise the decision maker on which misperceptions he should actively foster in order to improve the outcome of the game. In other words we could advise him on what type of player he should appear to be and which moves to take to give the impression that he was of that type.

The final question which remains to be answered is the question of what type of problem are we trying to solve with XDA. What problems should XDA be used for? We envision XDA as operating on the same range of problems as conventional Decision Analysis, that is, it would be used to analyze complex problems in business and government where a relatively large commitment of resources is planned. It would also be used in the same graduated manner as Decision Analysis, that is, a small "quick and dirty" analysis for small problems, perhaps only drawing the tree, and a large full scale analysis for major problems. Because XDA specifically includes Decision Analysis it should be able to do everything which Decision Analysis can now do. Because of its ability to explicitly handle competitive decision making and to deal accurately with decision makers who do not abide by Decision Analysis's axioms, XDA should handle many of these problems better than Decision Analysis.

B. APPLICATION ISSUES

This work should by no means be considered a "How To" handbook; it is a theoretical report on research in progress. However, we feel that the practical aspects of this type of work are of overriding importance and should be

considered at every turn. Thus we will consider two issues involving the practical application of XDA. We will not presume to describe one method as the "right" way, but will simply try to explain all sides of the issue.

1. To Prune or Not to Prune

This issue involves the interaction of the type of the opponents with the model of the game. The basic issue is whether or not the knowledge of the type of a player should be used to prune the game tree. For example, why not eliminate from the tree all successors to a decision which, given his type, the opponent will not make or does not know that he can make?

The pro-pruning argument is as follows. The trees of practical situations rapidly become large and unmanageable; the larger the tree the more difficult are the calculations to obtain a solution. Why not make use of the new information XDA provides us with to keep the tree under control, thus saving both time and money?

The con argument is two fold. One of the major complications of any competitive situation is the addition of the possibility of deception. By not including as many as possible of the available options on the tree we increase our susceptibility to deception and reduce our ability to practice it on our opponents. It may be to our distinct advantage to force the game into an area that one or more of our opponents considers impossible. The second consideration is that a tree which has been pruned on the basis of type of opponents makes sensitivity analysis on the basis of type invalid.

2. Modeling the Decision Maker as Multiple Players

Rex Brown [1978] argued, very persuasively, for the merits of treating future decisions by the decision maker as random events when conducting a decision analysis. The basic thrust of his argument is that no model can possibly allow, explicitly, for all contingencies such as unforeseen events or new information and that this inaccuracy in the model leads to inaccurate recommendations. Specifically, one tends to undervalue decisions which involve the gathering of information. He argues that it is more accurate to assess these uncertainties in some manner, even if not a strictly accurate one, rather than to ignore their existence. He proposes treating uncertainty about future information as uncertainty about future decisions. He claims that decision makers are more at ease with this format and more willing to make probabilistic assessments of their own future decisions than probabilistic assessments of unforeseen information.

XDA handles such situations by modeling the decision maker as more than one player. The first decision, the one that we must make now, as well as some subsequent decisions would be considered to be made by player 1, but other decisions by the decision maker would be considered to be made by other players. The decision maker could be represented by as many players as he has decision nodes.

The types of these players would be assessed by the decision maker in the same manner as the opponents'. Several possible causes of uncertainty

could be broken out and addressed separately as uncertainty about the type of the player. Uncertainty concerning totally unforeseen events can be retained as raw probabilities at the decision node.

The advantage of such a procedure is obvious, if you agree with Brown's argument. Even if you do not, we argue that, given that the probabilities involved can be assessed with reasonable accuracy, this form of model is inherently more accurate than a traditional one because it recognizes that no model can be completely accurate and attempts to take this inherent inaccuracy into account. It also allows greater flexibility in that it will allow us to model changing utility functions and similar phenomena directly.

The disadvantages are equally apparent, in the form of increased complexity of the model. This requires additional time from the decision maker in the assessment of the types of these additional players. It also requires additional effort in the solution procedure, as more than one type of player must be considered to solve subsequent decision maker decision nodes.

C. CONCLUSION

Our purpose is to help the decision maker analyze a competitive situation, using a methodology which is superior to both simple reasoning and using Decision Analysis when treating the opponent as a chance event. We propose the use of a substantial modification of Harsanyi's model's typology of players and a modification of the basic extensive form of Decision Analysis

to contain and present the information which the decision maker has about the game. We also propose the use of a modified form of rationalizable strategies and Bayesian updating to derive a solution from this information.

When more than one type of opponent is possible this procedure is mathematically equivalent to assigning probabilities directly to that opponent's decision. There are, however, several reasons why the probabilities assigned to opponent decisions in XDA should be superior to those arrived at by direct assessment. We follow the tradition of Decision Analysis in believing that small problems are easier for people to solve than big ones, and thus break the assessment of these probabilities into several steps. In addition we have brought in several psychological and mathematical constructs from Game Theory and psychology to aid in this assessment. We have also made an attempt, both in the Bayesian updating and the Hypergame sensitivity analysis, to explicitly come to grips with the infinite regress of expectations, and to include that factor in our assessment of these probabilities.

This methodology also holds promise for advising individuals who do not accept the axioms of Decision Analysis. While the method is not perfect and requires much more work to serve this purpose, it is superior to forcing those individuals to either accept advice based on expected utility, which is inaccurate for them, or to do without advice altogether.

We envision XDA as an analysis package, with the analyst being able to use whatever portions are necessary to solve the problem at hand. That goal

has not been reached with this work, but progress has been made. There is a great deal more work to be done: expanding and stating with mathematical rigor our modified form of rationalizable strategies, finding more than two whole game solution concepts, more thoroughly categorizing the typology of players, more closely and rigorously examining the infinite regress of perception and expectation both as it applies to Hypergame sensitivity analysis and to Bayesian updating of priors, and finally developing and computerizing the display techniques that will be required to successfully implement this methodology. All in all we feel that XDA has the potential to be an extremely powerful tool to improve the quality of decisions made in competitive situations. At least part of that potential has been realized in this work.

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